

# Algebra

by

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To Peter A. Cassileth and to the memories of Stanley M.K. Chung and Ralph C. Marcove, physicians and friends all. Being mortal and denied the gift of life, he gives and they gave the next best thing: Time

To Anne, Mia, Philippe and Sylvie



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