Refinement Through Restraint: Bringing Down the Cost of Verification

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\textbf{Abstract}

We present a framework aimed at significantly reducing the cost of verifying certain classes of systems software, such as file systems. Our framework allows for equational reasoning about systems code written in our new language, \textsc{Cogen}. \textsc{Cogen} is a restricted, polymorphic, higher-order, and purely functional language with linear types and without the need for a trusted runtime or garbage collector. Linear types allow us to assign two semantics to the language: one imperative, suitable for efficient C code generation; and one functional, suitable for equational reasoning and verification. As \textsc{Cogen} is a restricted language, it is designed to easily interoperate with existing C functions and to connect to existing C verification frameworks.

Our framework is based on \textit{certifying compilation}: For a well-typed \textsc{Cogen} program, our compiler produces C code, a high-level shallow embedding of its semantics in Isabelle/HOL, and a proof that the C code correctly refines this embedding. Thus one can reason about the full semantics of real-world systems code productively and equationally, while retaining the interoperability and leanness of C. The compiler certificate is a series of language-level proofs and per-program translation validation phases, combined into one coherent top-level theorem in Isabelle/HOL.

\textbf{Categories and Subject Descriptors} \quad D.2.4 [Software Engineering]: Software / Program Verification—Formal methods; \quad F.3.2 [Logics and Meanings of Programs]: Semantics of Programming Languages; \quad D.3.2 [Programming Languages]: Language Classification—Applicative (functional) languages

\textbf{Keywords} \quad verification; semantics; linear types; programming languages; file systems; Isabelle/HOL

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\textbf{1. Introduction}

Imagine writing low-level systems code in a purely functional language and then reasoning about this code equationally and productively in an interactive theorem prover. Imagine doing this without the need for a trusted compiler, runtime or garbage collector and letting this code interoperate with native C parts of the system, including your own efficiently implemented and formally verified additional data types and operations.

Our verification framework achieves this goal through the certifying compiler of \textsc{Cogen}: a high-level, pure, polymorphic, functional language with linear types, specifically designed for certain classes of systems code such as file systems. For a given well-typed \textsc{Cogen} program, the compiler will produce a high-level shallow embedding of the program’s semantics in Isabelle/HOL \cite{Nipkow and Klein 2014}, and a proof that connects this shallow embedding to the compiler generated C code: any property proved of the shallow embedding is guaranteed to hold for the generated C.

The compilation target is C, because C is the language in which most existing systems code is written, and because with the advent of tools like CompCert \cite{Leroy 2006, 2009} and gcc translation validation \cite{Sewell et al. 2013}, large subsets of C now have a formalised semantics and an existing formal verification infrastructure.

If C is so great, why not verify C systems code directly? After all, there is an ever growing list of successes \cite{Beringer et al. 2015, Gu et al. 2015, Klein et al. 2009, 2014} in this space. The reason is simple: verification of manually written C programs remains expensive. Just as high-level languages increase programmer productivity, they should also increase verification productivity. \textsc{Cogen} is specifically designed with a verification-friendly high-level semantics. This makes the difference between imperative and functional verification. It is pointer fiddling and undefined behaviour guards in C versus abstract functional objects and equations in \textsc{Cogen}. An imperative VCG \cite{Dijkstra 1997} for C must overwhelm the prover with detail, while the abstraction and type system of \textsc{Cogen} enable the use of far stronger existing automation for high-level proofs.

The state of the art for certifying compilation of functional languages is CakeML \cite{Kumar et al. 2014}, which covers an entire ML dialect. \textsc{Cogen} is targeted at a substantially different point in the design space. CakeML includes a verified runtime and garbage collector, while \textsc{Cogen} works hard to avoid these so it can be applicable to low-level embedded systems code. CakeML covers full Turing-complete ML with complex, stateful semantics, which works well for code written in theorem provers. \textsc{Cogen} is a restricted language of total functions with intentionally simple, pure

\textsuperscript{1} Mind the potential logical gap between our C parser’s C semantics \cite{Tuch et al. 2007} and that of CompCert.
semantics that are easy to reason about equationally. CakeML is great for application code; Cogent is great for systems code, especially layered systems code with minimal sharing such as the control code of file systems or network protocol stacks. Cogent is not designed for systems code with closely-coupled, cross-cutting sharing, such as microkernels.

Cogent’s main restrictions are the (purposeful) lack of recursion and iteration and its linear type system. The former ensures totality, which is important for both systems code correctness as well as for a simple shallow representation in higher-order logic. The latter is important for safe memory management and for enabling a transition from an imperative C-style semantics, suitable for code generation, to a functional semantics, suitable for equational reasoning and verification.

Even in the restricted target domains of Cogent, real programs will contain some amount of iteration. This is where Cogent’s integrated foreign function interface (FFI) comes in: the engineer provides her own verified data types and iterator interfaces in C and uses them seamlessly in Cogent, including in formal reasoning. Our framework guarantees that the verification of combined C-Cogent code bases has no room for unsoundness.

Cogent is restricted, but it is by no means a toy language. We have previously used Cogent to successfully implement two efficient full-scale Linux file systems [Amani et al. 2016] — the standard Linux ext2 and the BilbyFs Flash file system [Keller et al. 2013], and prove two core functional correctness properties of BilbyFs. This illustrates that Cogent is suitable both for implementation and proofs, dramatically reducing the cost of verifying correctness of practical file systems. These case studies are beneficial in their own right, as file systems constitute the second largest proportion of OS code, and have among the highest density of faults [Palix et al. 2011]. The benefits of this language-based approach for file system verification were conjectured by Keller et al. [2013] and are confirmed by our work.

Our main contribution is the framework for significantly reducing the cost of formal verification for important classes of systems code, using this language-based approach for automatically co-generating code and proofs. Specifically, our framework relies on the following technical contributions:

- the Cogent language, its formal semantics and its certifying compiler;
- the formal machine-checked proof for switching from imperative update semantics to functional value semantics for a full-featured functional language, justified by linear types (§3). We build upon well-known theoretical results about linear types, accounting for pointers and heap allocation;
- the formal description of the assumptions required for C code imported via an FFI to maintain the guarantees of the linear type system (§3.3.2);
- the top-level compiler certificate (§4.1); and
- the verification stages that make up the correctness theorem (§4), including automated refinement calculi, formally verified type checking, A-normalisation, and monomorphisation. One of these stages is presented in detail elsewhere [Rizkallah et al. 2016], but summarised in §4.3.

Our case study file system implementations (presented in §5) demonstrate that our approach supports verified systems programming at scale: providing simple, easy to reason about shallow embeddings for large-scale software systems by pushing the design and associated metatheory of linear type systems to the scale of practical languages. The compiler, Isabelle proofs, and file systems are available online.²

2. Overview

Our certifying compiler constructs a chain of proofs relating Cogent to efficient C code, such that a proof engineer can reason equationally about its semantics in Isabelle/HOL and apply the compiler theorem to derive properties about the generated C code. Formally, the certificate theorem is a refinement statement between the shallow embedding and the C code. This generated C code falls into the subset that can be compiled by CompCert, or validated by the gcc translation validation tool of Sewell et al. [2013], whose theorem would compose directly with our compiler certificate.³

Shallow embeddings are nice for humans, but do not provide much syntactic structure for constructing a compiler theorem. Therefore the compiler also generates a deep embedding for each Cogent program to use in the internal proof chain. There are two semantics for this deep embedding: (1) a formal functional value semantics where programs evaluate to values and (2) a formal imperative update semantics where programs manipulate references to mutable global state.

Fig. 1 shows an overview of the program representations generated by the compiler and the break-down of the automatic refinement proof that makes up the compiler certificate. The program representations are, from the bottom of Fig. 1: the C code, the semantics of the C code expressed in Isabelle/Simpl [Schirmer 2006], the same expressed as a monadic functional program [Greenaway et al. 2012, 2014], a monomorphic A-normal [Sabry and Felleisen 1992] deep embedding of the Cogent program, a polymorphic A-normal deep embedding of the same, an A-normal shallow embedding, and finally a ‘near’ shallow embedding of the Cogent program that is syntactically close to the Cogent source code. Several theorems


³ At the time of writing, Cogent’s occasionally larger stack frames lead to gcc emitting memcpy() calls that, while conceptually straightforward to handle, the translation validator does not yet cover.
rely on the COGENT program being well-typed, which we prove automatically using type inference information from the compiler.

The solid arrows on the right-hand side of the figure represent refinement proofs and the arrow labels correspond to the numbers in the following description. The only arrow that is not formally verified is the one crossing from C code into Isabelle/HOL at the bottom of Fig. 1 — this is the C parser [Tuch et al. 2007], which is a mature verification tool used in a number of large-scale verifications [Klein et al. 2009]. As mentioned, it could be checked by translation validation.

We outline each intermediate theorem, starting with Simpl at the bottom. For well-typed COGENT programs, we prove:

1. The Simpl code produced by the C parser corresponds to a monadic representation of the C code. The proof is generated using an adjusted version of AutoCorres [Greenaway et al. 2012].
2. The monadic code terminates and is a refinement of the update semantics of the monomorphic COGENT deep embedding [Rizkallah et al. 2016].
3. If a COGENT deep embedding evaluates in the update semantics, it evaluates to the same result in the value semantics. We mechanise this consequence of linear type systems, significantly extending known theoretical results to accommodate heap data structures, on a real full-scale language.
4. If a monomorphic COGENT deep embedding evaluates in the value semantics then the polymorphic deep embedding evaluates equivalently in the value semantics.
5. If the polymorphic COGENT deep embedding evaluates in the value semantics then the COGENT shallow embedding evaluates to a corresponding shallow Isabelle/HOL value.
6. The shallow embedding is (extensionally) equal in Isabelle/HOL to a neater shallow embedding, more convenient for human reasoning. The neat shallow embedding corresponds to the COGENT code before being A-normalised.

In §4 we define in more detail the relations that formally link the values (and states, when applicable) that these programs evaluate to. Steps 1 and 4 are general properties about the language and we therefore prove them manually once and for all. Steps 2, 3, and 5 are generated automatically for every program. The proof for step 6 is generated by AutoCorres. For steps 2 and 5 we define compositional refinement calculi that ease the automation of these proofs. Step 6, the correctness of A-normalisation, is straightforward to prove via rewriting because at this stage we can already use equational reasoning.

The refinement proofs state that every behaviour exhibited by the C code can also be exhibited by the COGENT code and, furthermore, that the C code is always well-defined, including that e.g. the generated C code never dereferences a null pointer, and never causes signed overflow. It also implies that the generated C code is type-safe and memory-safe, meaning the code will never try to dereference an invalid pointer, or try to dereference two aliasing pointers of incompatible types. In conjunction with the COGENT typing proofs, generated by the COGENT compiler for the input program, we get additional guarantees that the generated code handles all error cases, is free of memory leaks, and never double-frees a pointer. These points will be formally established in §3 and §4.

We previously demonstrated [Amani et al. 2016] how to reason on top of the COGENT formal specification in Isabelle/HOL (arrow 7 in Fig. 1), leveraging COGENT’s purely functional semantics. Reasoning here is simpler than reasoning on C code directly because the COGENT semantics is represented as pure functions in HOL, rather that complex, stateful embeddings. This purely functional representation makes it possible to reason equational about COGENT code via Isabelle’s powerful rewriting engine. If desired, one could formalise domain-specific program logics, such as the Crash Hoare Logic of Chen et al. [2015], on top of our generated COGENT specification. This would enable further improvements to verification productivity by simplifying reasoning about domain-specific properties.

3. COGENT Language

Fig. 2 contains an excerpt of a COGENT ext2 implementation, which demonstrates the use of most language features. Line 1 shows the COGENT side of the FFI. It declares an abstract COGENT data type ExSt, implemented in C. Line 2 shows a parametric abstract type, and line 9 a corresponding abstract function uarray_create(), also implemented in C. uarray_create() is polymorphic, with a kind constraint E (see §3.1) on type argument a.

```
| type ExSt  
| type UArray a  
| type Opt a = None () | Some a  
| type Acc = (ExSt, PstSt, VfsInode)  
| type Cnt = (UArray Node, ...)  
| (U32, Node, Acc, U32, UArray Node) -> (Node, Acc))  
| uarray_create: all (a :: E). (ExSt, U32)  
| -> <Success (ExSt, UArray a) | Err ExSt>  
| ext2_free_branch: (U32, Node, Acc, U32)  
| -> (Node, Acc, <Expd Cnt | Iter ()>)  
| ext2_free_branch (depth,nd,(ex,fs,inode),mdep) =  
| if depth < mdep  
| then uarray_create[Node] (ex,nd.to-nd.fr) !nd  
| | Success (ex, children) =>  
| | let nd_t = { mbuf } = nd  
| | and (children, (ex, inode, .., mbuf)) =  
| | uarray_map_no_break (#  
| | arr = children,  
| | f = ext2_free_branch_entry,  
| | acc = (ex, inode, node_t.fr, mbuf),  
| | ... }) !nd_t  
| | and nd = nd_t { mbuf }  
| | in (nd, (ex, fs, inode),  
| | Expd (children, ext2_free_branch_cleanup))  
| | | Err ex -> (nd, (ex,fs,inode), Iter ())  
| | else ...  
```

Figure 2: COGENT example
surface syntax for Cogent’s linear take construct (see §3.2.3), accessing and binding the mbuf field of nd to the name mbuf (punning as in Haskell), as well as binding the rest of the record to the name nd.x. The linear type system tracks that the field mbuf is logically absent in nd.x. It also tracks that nd on line 18 was used, so cannot be accessed again. Thus the programmer is safe to bind a new object to the same name nd (on line 25) without worrying about name shadowing. Line 25 shows surface syntax for put, the dual to take, which re-establishes the mbuf fields in the example.

### 3.1 Types and Kinding

Wadler [1990] first noted that linear types can be used as a way to safely model mutable state and similar effects while maintaining a purely functional semantics. Hofmann [2000] later proved Wadler’s intuition by showing that, for a linear language, imperative C code without heap-allocated data structures can implement a simple set-theoretic semantics.

Many languages such as Rust [Rust] and Vault [DeLine and Fähndrich 2001] use linear types to manage resources such as memory and eliminate the need for run-time support such as garbage collection.

We use linear types for both of these reasons: we do not require a garbage collector, and we assign to Cogent programs an equational, purely functional semantics implemented via mutable state internally. Unlike Hofmann, our mechanised proof of the correspondence between these two semantics takes heap-allocated data structures into account, using our heap footprint annotation technique (§3.3.1).

The type structure and associated syntax of Cogent is presented in Fig. 3. Our type system is polymorphic, but we restrict this polymorphism to be rank-1 and predicative, in the style of ML, to permit easy implementation by specialisation with minimal performance penalty.

To ease implementation, and to eliminate any direct dependency on a heap allocator, we require that all functions be defined on the top-level, disallowing closures. Any top-level function can be shared freely, as they cannot capture any local variables, let alone linear ones.

We include a set of primitive integer types (U8, U16, etc.). Records \( \langle \Gamma \vdash \tau \mid \gamma \rangle \) m comprise (1) a sequence of fields \( f :: \gamma \), where \( \gamma \) is the type on an inaccessible field, and (2) a mode \( m \) (see §3.2.3 and §3.1.1 for a more detailed description). We also have polymorphic variants \( \langle \tau, \gamma \rangle \), a generalised sum type in the style of OCaml, the mechanics of which are briefly described in §3.2.2. Abstract types \( \tau, \gamma \) m are also parametrised by modes. We omit product types from this presentation; they are desugared into unboxed records.

Similarly to other polymorphic, substructural type systems such as \( \lambda_{\text{URAL}} \) [Ahmed et al. 2005] and System \( \mathcal{F} \) [Mazurak et al. 2010], we use kinds to determine if a type may be freely shared or discarded. Kinds in Cogent are sets of permissions, denoting whether a variable of that type may be discarded without being used (D), shared freely and used multiple times (S), or safely bound in a let! expression (E). A linear type, values of which must be used exactly once, has a kind that excludes D and S, and so forbids it being discarded or shared. We discuss let! expressions in §3.1.2.

We explicitly represent the context operations of weakening and contraction, normally relegated to structural rules, as explicit judgements: \( \Delta \vdash \tau \vdash \kappa \vdash \Gamma \) for weakening (discarding assumptions) and \( \Delta \vdash \Gamma \vdash \kappa \vdash \Gamma \) for contraction (duplicating them). The rules for these judgements are presented in Fig. 3. For a typing assumption to be discarded (respectively duplicated), the type must have kind \( \{ \text{D} \} \) (resp. \( \{ \text{S} \} \)).

The full kinding rules for the types of Cogent are given in Fig. 4. Basic types such as \( \text{id} \) or \( \text{U8} \), as well as functions, are simply passed by value and do not contain any heap references, so they may be given any kind. Kinding for structures and abstract functions is discussed shortly in §3.1.1.

A type may have multiple kinds, as a nonlinear type assumption may be used linearly, never being shared and being used exactly once. Therefore, a type with a permissive kind, such as \( \{ \text{D}, \text{S} \} \), would be an acceptable instantiation of a type variable of kind \( \emptyset \), as we are free to waive permissions that are included in a kind. We can prove formally by straightforward rule induction:

**Lemma 1** (Waiving rights). If \( \Delta \vdash \tau \vdash \kappa \) and \( \kappa' \subseteq \kappa \), then \( \Delta \vdash \tau \vdash \kappa' \).

This result allows for a simple kind-checking algorithm, not immediately apparent from the rules. For example, the maximal kind of an unboxed structure with two fields of type \( \tau_1 \) and \( \tau_2 \) respectively can be computed by taking the intersection of the computed maximal kinds of \( \tau_1 \) and \( \tau_2 \). This result ensures that this intersection is also a valid kind for \( \tau_1 \) and \( \tau_2 \).

### 3.1.1 Kinding for Records and Abstract Types

Recall that Cogent may be extended with abstract types, implemented in C, which we write as \( \langle \Gamma \rangle m \) in our formalisation. We allow abstract types to take any number of type parameters \( \tau_i \), where each specific instance corresponds to a distinct C type. For example, a List abstract type, parameterised by its element type, would correspond to a family of List types, each specialised to a particular concrete element type. Because the implementations of these types are user supplied, the user is free to specialise implementations based on these type parameters, for example representing an array of boolean values as a bitstring, so long as they can show that every different operation implementation is a refinement of the same user-supplied CDSL semantics for that operation.

Values of abstract types may be represented by references to heap data structures. Specifically, an abstract type or structure is stored on the heap when its associated storage mode \( m \) is not “u”. For boxed records and abstract types, the storage mode distin-

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**Figure 3:** Type Structure of Cogent & structural context operations

<table>
<thead>
<tr>
<th>prim. types</th>
<th>( t ) ::= U8</th>
<th>U16</th>
<th>U32</th>
<th>U64</th>
<th>Bool</th>
</tr>
</thead>
<tbody>
<tr>
<td>types ( \tau, \rho )</td>
<td>( \alpha \mid \alpha' \mid () )</td>
<td>( t \mid T \mid \tau \mid m \mid \tau \rightarrow \rho )</td>
<td>( \langle \Omega \rangle \tau \mid \langle \Gamma \rangle m )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>field types ( \tau' )</td>
<td>( \tau \mid \sigma )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>permissions ( \mathcal{P} )</td>
<td>{D, S, E}</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>kinds ( \kappa )</td>
<td>( \subseteq \mathcal{P} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>polytypes ( \pi )</td>
<td>( \forall (\alpha :: \kappa \times \alpha) \rightarrow \tau )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>modes ( m )</td>
<td>( \tau \mid \omega \mid u )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>type variables</td>
<td>( \exists \alpha )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>abs. type names</td>
<td>( \exists T, U )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>kind context ( \Delta )</td>
<td>( \odot \alpha :: \kappa \times \alpha )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>type context ( \Gamma )</td>
<td>( \times \tau )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Figure 3:** Type Structure of Cogent & structural context operations
guishes between those that are “w” vs. “r”. The same is true for
record types, written \( \{i :: \tau_i\} m \), which are discussed in more
detail in §3.2.3.

The storage mode \( m \) affects the maximal kind that can be as-
signed to the type. For example, an unboxed structure with two
components of type \( \text{U8} \) is freely shareable, but if the structure is in-
stead stored on the heap, then a writable reference to that structure
must be linear. Thus, the kind given to such references has the “w”
mode, whose kind is \( |E| \), thereby preventing such a reference from
being assigned a nonlinear kind such as \( |D, S| \).

### 3.1.2 Kinding and bang

We allow linear values to be shared read-only in a limited scope,
an idea first explored by Wadler [1990]. This is useful for practical
programming in a language with linear types, as it makes our types
more informative. For example, to write a function to determine the
size of a (linear) buffer object, a naive approach would be to write a
function:

\[
\text{size} : \text{Buf} \rightarrow \text{U32} \times \text{Buf}
\]

This function has a cumbersome additional return value just so
that the linear argument is not discarded. Further, the type above
does not express the fact that the input buffer and output buffer are
identical — this would need to be established by additional proof.
To address this problem, we include a type operator \( \text{bang}() \), in
the style of Wadler’s ! operator, which changes all writable modes in
a type to read-only ones. The full definition of \( \text{bang}() \) is in Fig. 4.
We can therefore write the type of our function as:

\[
\text{size} : \text{bang}(\text{Buf}) \rightarrow \text{U32}
\]

For any valid type \( \tau \), the kind of \( \text{bang}(\tau) \) will be nonlinear, which
means that our \text{size} function no longer needs to be encumbered by
the extra return value. This kinding result is formally stated as:

**Lemma 2 (Kinding for bang()).** For any type \( \tau \), if \( \Delta \vdash \tau \vdash_k \kappa \) then
\( \Delta \vdash \text{bang}(\tau) \vdash_k \text{bang}(\kappa) \).

To integrate this type operator with parametric polymorphism, we
model our solution on Odersky’s Observer types [Odersky 1992],
and tag type variables that have been made read only, using the
syntax \( \alpha! \). Whenever a variable \( \alpha \) is instantiated to some concrete
type \( \tau \), we also replace \( \alpha! \) with \( \text{bang}(\tau) \). The lemma above ensures
that our kinding rule for such tagged variables is sound, and enables
us to prove the following:

**Lemma 3 (Type instantiation preserves kinds).** For any type \( \tau \),
\( \alpha_i (\kappa_i) \vdash \tau \vdash_k \kappa \) implies \( \Delta \vdash \tau |\{\alpha_i/\alpha\}| \vdash_k \kappa \) when, for each \( i \),
\( \Delta \vdash \rho_i (\kappa_i) \).

### 3.2 Expressions and Typing

We only document the core language of Cogent in Fig. 5 to which
the surface syntax is desugared, leaving out the richer surface
syntax due to space constraints.

Fig. 6 shows the typing rules for Cogent expressions. Many of
these are standard for any linear type system. We will discuss here
the rules for let!, where we have taken a slightly different approach
to established literature, and the rules for the extensions we have
made to the type system, such as variants and record types.
3.2.1 Typing for let!

On the expression level, the programmer can use `let!` expressions, in the style of [Wadler 1990], to temporarily convert variables of linear types to their read-only equivalents, allowing them to be freely shared. In this example, we wish to copy a buffer `b1` onto a buffer `b2` only when `b2` will fit inside `b1`.

```
let!(b1, b2) ok = (size(b2) < size(b1)) in
if ok then copy(b1, b2) else ...
```

Note that even though `b1` and `b2` are used multiple times, they are only used once in a linear context. Inside the `let!` binding, they have been made temporarily nonlinear. Our kind system ensures these read-only, shareable references inside `let!` bindings cannot “escape” into the outside context. For example, the expression `let!(b) b' = b in copy(b, b')` would violate the invariants of the linear type system, and run the purely functional abstraction that linear types allow, as both `b` and `b'` would refer to the same object, and a destructive update to `b` would change the shareable `b'`.

We are able to use the existing kind system to handle these safety checks with the inclusion of the `E` permission, for Escapable, which indicates that the type may be safely returned from within a `let!`. We ensure, via the typing rules of Fig. 6, that the left hand side of the binding (ok in the example) has the `E` permission, which excludes temporarily nonlinear references via `bang(·)` (see Fig. 4).

3.2.2 Typing for Variants

A variant type `(C, τ)` is a generalised sum type, where each alternative is distinguished by a unique data constructor `C`. The order in which the constructors appear in the type is not important. One can create a variant type with a single alternative simply by invoking the `singleton` constructor.

```
let (C, τ) = singleton
```

The original value of 255 can be retrieved using the `esac` construct. The set of alternatives is enlarged by using `promote` expressions that are automatically inserted by the type-checker of the surface language, which uses subtyping to infer the type of a given variant.

A similar trick is used for numeric literals and `cast`.

In order to pattern match on a variant, we provide a `case` construct that attempts to match against one constructor. If the constructor does not match, it is `removed` from the type and the reduced type is provided to the `else` branch. In this way, a traditional multi-way pattern match can be desugared by nesting:

```
case x of
  A a → e_a
  B b → e_b
  C c → e_c
  else x' → let c = esac x'' in e_c
```

Note that because the typing rule for `esac` only applies when only one alternative remains, our pattern matching is necessarily total.

<table>
<thead>
<tr>
<th>$\Delta \vdash e : \tau$</th>
<th>$\Delta, \Gamma \vdash e : (\text{let})$</th>
<th>$\Delta, \Gamma \vdash e : (\text{case})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta, \Gamma \vdash e : \tau$</td>
<td>$\Delta, \Gamma \vdash e : (\text{let})$</td>
<td>$\Delta, \Gamma \vdash e : (\text{case})$</td>
</tr>
<tr>
<td>$\Delta, \Gamma \vdash e : (\text{case})$</td>
<td>$\Delta, \Gamma \vdash e : (\text{case})$</td>
<td>$\Delta, \Gamma \vdash e : (\text{case})$</td>
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<td>$\Delta, \Gamma \vdash e : (\text{case})$</td>
<td>$\Delta, \Gamma \vdash e : (\text{case})$</td>
<td>$\Delta, \Gamma \vdash e : (\text{case})$</td>
</tr>
</tbody>
</table>

Figure 6: Typing rules for Cogent
3.2.3 Typing for Records

Some care is needed to reconcile record types and linear types. Assume that Object is a type synonym for an (unboxed) record type containing an integer and two (linear) buffers.

\[ \text{Object} = \{\text{size} :: \text{U}32, \text{b}_1 :: \text{Buf}, \text{b}_2 :: \text{Buf}\} u \]

Let us say we want to extract the field \( b_1 \) from an Object. If we extract just a single Buf, we have implicitly discarded the other buffer \( b_2 \). But, we can’t return the entire Object along with the Buf, as this would introduce aliasing. Our solution is to return along with the Buf an Object where the field \( b_1 \) cannot be extracted again, and reflect this in the field’s type, written as \( b_1 :: \text{Buf}^2 \). This field extractor, whose general form is \( \text{take} x \{f = y\} = e_1 \), in \( e_2 \), operates as follows: given a record \( e_1 \), it binds the field \( f \) of \( e_1 \) to the variable \( y \), and the new record to the variable \( x \) in \( e_2 \). Unless the type of the field \( f \) has kind \([\text{S}]\), that field will be marked as unavailable, or taken, in the type of the new record \( x \).

Conversely, we also introduce a \( \text{put} \) operation, which, given a record with a taken field, allows a new value to be supplied in its place. The expression \( \text{put} e_1.f = e_2 \) returns the record in \( e_1 \) where the field \( f \) has been replaced with the result of \( e_2 \). Unless the type of the field \( f \) has kind \([\text{D}]\), that field must already be taken, to avoid accidentally destroying our only reference to a linear resource.

Unboxed records can be created using a simple struct literal \( [l_i = e_i] \). We also allow records to be stored on the heap to minimize unnecessary copying, as unboxed records are passed by value. These boxed records are created by invoking an externally-defined C allocator function. For these allocation functions, it is important to specialise code to avoid paying the performance penalties of unnecessary copying, as unboxed records are passed by value. Assuming that polymorphism is to define a correspondence relation between update semantics and value semantics, evaluation implies a corresponding value semantics evaluation.

3.2.4 Type Specialisation

As mentioned earlier, we implement parametric polymorphism by specialising code to avoid paying the performance penalties of other approaches such as boxing. This means that polymorphism in our language is restricted to predicative rank-1 quantifiers.

This allows us to specify dynamic objects, such as our value typing relations (see §3.3.1) and our dynamic semantics (see §3.3), in terms of simple monomorphic types, without type variables. Thus, in order to evaluate a polymorphic program, each type variable must first be instantiated to a monomorphic type. We show that typing of the instantiated program follows from the typing of the polymorphic program, if the type instantiation used matches the kinds of the type variables.

Lemma 4 (Type specialisation). \( \alpha_i : \cdot \kappa_i \in \Gamma \vdash e : \tau \) implies \( \Delta | [\beta_i / \kappa_i] \vdash e | [\beta_i / \kappa_i] : \tau | [\beta_i / \kappa_i] \) when, for each \( i \), \( \Delta \vdash \rho_i : \cdot \kappa_i \).

The above lemma is sufficient to show the monomorphic instantiation case, by setting \( \Delta = e \) (the empty context). This lemma is a key ingredient for the refinement link between polymorphic and monomorphic deep embeddings (See §4.4).

3.3 Dynamic Semantics

Fig. 8 includes most of the evaluation rules for the value semantics of Coq, as a big-step relation \( V \vdash e \downarrow v \), \( v \) from expressions to their values. These values are documented in Fig. 7. This semantics is typical of a purely functional language, by design, and we automatically produce a purely functional shallow embedding from it.

As functions must be defined on the top level, our function values \( \langle\langle x.e\rangle\rangle \) consist only of an unevaluated expression, evaluated when the function is applied. Abstract function values, written \( \langle\langle \text{abs}.f | \tau \rangle \rangle \), are instead passed indirectly, as a pair of the function name and a list of the types used to instantiate type variables. When an abstract function value \( \langle\langle \text{abs}.f | \tau \rangle \rangle \) is applied, the user-supplied semantics \( \llbracket f \rrbracket \) is invoked, which is just a function from input to output value.

The update semantics, by contrast, is rather imperative. An excerpt is presented in Fig. 8, with associated definitions in Fig. 7. This semantics is an evaluation semantics, written \( U \vdash e | \mu \downarrow_u \). Values in the update semantics may now be pointers, written \( p \), to values in a mutable store or heap \( \mu \). This mutable store is modeled as a partial function from a pointer to an update semantics value.

Most rules in the update semantics only differ from the value semantics in that they thread the heap \( \mu \) through the program evaluation, and so many of those rules have been omitted. However, key differences arise in the treatment of records and of abstract types, which may now be represented as boxed structures, stored on the heap. In particular, note that the rule \( \text{Up}\vdash \text{ update} \) destructively updates the heap, instead of creating a new record value, and the semantics of abstract functions \( \llbracket f \rrbracket \) may also modify the heap.

3.3.1 Update-Value Refinement and Type Preservation

To show the update semantics is a refinement of the value semantics, we must exploit the information given by Coq\textsubscript{ann}’s linear type system. A typical refinement approach to relate the two semantics is to define a correspondence relation between update semantics states and value semantics values, and show that an update semantics evaluation implies a corresponding value semantics evaluation. However, such a statement is not true if aliasing exists, as a destructive update (from, say, \( \text{put} \)) would result in multiple values being changed in the update semantics but not necessarily in the value semantics. As our type system forbids aliasing of writable references,
we must include this information in our correspondence relation. Written as $u \mid \mu : v : \tau \mid r \ast w$, this relation states that the update semantics value $u$ with heap $\mu$ corresponds to the value semantics value $v$, which both have the type $\tau$. Unlike prior work [Hofmann 2000], we account for the heap by annotating this relation with the sets $r$ and $w$, which contain all pointers accessible (transitively) from the value $u$ that are read-only and writable respectively. We call these pointer sets the heap footprint of the value. By annotating the correspondence relation with the heap footprint, we can encode the uniqueness properties ensured by linear types as explicit non-aliasing constraints in the rules, given in Fig. 9. Read-only pointers may alias other read-only pointers, but writable pointers do not alias any other pointer, whether read-only or writable.

Because our correspondence relation includes types, it naturally implies a value typing relation for both semantics (written $v : \tau$) and update semantics (written $u \mid \mu : \tau : \{\mu \mid 1 \ast w\}$). In fact, the rules for correspondence can be derived from the rules of Fig. 10 simply by erasing either the value semantics parts (highlighted like this) or the update semantics parts (highlighted like this). As we ultimately prove preservation for this correspondence relation across evaluation, this same erasure strategy can be applied to our proofs to produce a type preservation proof for either semantics.

**Dealing with mutable state** We define a framing relation which specifies exactly how evaluation may affect the mutable heap $\mu$. Given an input set of writable pointers $w_t$, an input heap $\mu_t$, an output set of pointers $w_o$ and an output heap $\mu_o$, the relation, written $w_t \mid \mu_t, \text{frame} w_o \mid \mu_o$, ensures three properties for any pointer $p$:

- **Inertia** If $p \notin w_t \cup w_o$, then $\mu_o(p) = \mu_t(p)$.
- **Leak freedom** If $p \in w_t$ and $p \notin w_o$, then $\mu_o(p) = \bot$.
- **Fresh allocation** If $p \notin w_t$ and $p \in w_o$, then $\mu_o(p) = \bot$.

Framing implies that our correspondence relation, for both values and environments, is unaffected by updates to parts of the heap not mentioned in the heap footprint:

**Lemma 5** (Unrelated updates). Assume two unrelated pointer sets $w_t \cap w_o = \emptyset$ and that $w_t \mid \mu_t, \text{frame} w_o \mid \mu_o$, then

- If $u \mid \mu : v : \tau \mid r \ast w$ then $u \mid \mu : v : \tau \mid r' \ast w$ and $w \cap w_o = \emptyset$.
- If $U \mid \mu : V : \Gamma \mid r \ast w$ then $U \mid \mu : V : \Gamma \mid r \ast w$ and $W \cap w_o = \emptyset$.

We are now able to prove refinement between the value and the update semantics. We first prove that the correspondence relation is preserved when both semantics evaluate from corresponding environments. By erasing one semantics, this becomes a type preservation theorem for the other. Full details of the proof are available in our Isabelle/HOL formalisation.

**Theorem 1** (Preservation of types and correspondence). For any program $e$ where $\Delta; \Gamma \vdash e : \tau$, if $U \mid \mu : V : \Gamma \mid r \ast w$ and $V \vdash e : \tau \mid r' \ast w'$ then there exists $r' \subseteq r$ and $w'$ such that $u \mid \mu \mid \tau : v : \tau'$ and $w \cap w_o = \emptyset$.

To prove refinement, we must show that every evaluation on the concrete update semantics has a corresponding evaluation in the abstract value semantics. While Theorem 1 gets us most of the way there, we still need to prove that the value semantics can evaluate whenever the update semantics does.

**Theorem 2** (Upward-propagation of evaluation). For any program $e$ where $\Delta; \Gamma \vdash e : \tau$, if $U \mid \mu : V : \Gamma \mid r \ast w$ and $U \vdash e \mid \mu : u \mid \mu'$, then there exists a $v$ such that $V \vdash e \mid \mu : v$.

Composing this lemma and Theorem 1, we can now easily prove our desired refinement statement.

**Theorem 3** (Value ⇒ Update refinement). For any program $e$ where $\Delta; \Gamma \vdash e : \tau$, if $U \mid \mu : V : \Gamma \mid r \ast w$ and $U \vdash e \mid \mu : u \mid \mu'$.
then there exists a value \( v \) and point sets \( r' \subseteq r \) and \( w' \) such that \( V \vdash e \Downarrow v \), and \( u \mid \mu' : \tau : [r' \cup w'] \) and \( w \mid \mu \text{ frame } w' \mid \mu' \).

### 3.3.2 FFI requirements

Each of the above theorems makes certain assumptions about the the semantics given to abstract functions, \( [\cdot] \), and \( \llbracket \cdot \rrbracket \). Specifically, we assume the preservation theorem above for these functions: for any abstract function \( f \) of type \( \tau \to \rho \), if the argument values \( u \) and \( v \) correspond and are well typed i.e. \( u \mid \mu \vdash v : \tau : [r \cup w] \) and \( [f] \llbracket (u, \mu) \rrbracket = ([f] \llbracket (u, \mu') \rrbracket ) \) and \( [f], v = v' \), then \( u' \mid \mu' \vdash v' : \rho : [r' \cup w'] \) for some \( r' \subseteq r \) and \( w' \) such that \( w \mid \text{ frame } w' \mid \mu' \). The well-typedness requirement ensures that the returned value does not include internal aliasing, and the \text{frame} requirements ensure that the abstract function does not manipulate any state other than values passed into it. These requirements are necessary to integrate C code with Coq's linear type system. In order to show refinement, we must also assume that abstract functions evaluate in the value semantics whenever they evaluate in the update semantics, as in Theorem 2.

### 4. Compiler Certificate

This section describes each of the proof steps that make up the compiler certificate, depicted in Fig. 1.

#### 4.1 Top-Level Theorem

The top-level theorem is the overall certificate emitted by the compiler. We say a C program correctly implements its Coq'st shallow embedding if: (i) it terminates with defined execution; and (ii) assuming that the initial C and Coq'st stores are related and the program inputs are related, then their outputs are related.

In other words, the compiler theorem states that a value relation \('V'\) between a result \( v\text{a} \) of a C program \( p\text{a} \) and a shallow embedding \( s \), that relies on the intermediate update and value semantics results. The relation in \( \S 3.3.3 \) depends on a Coq'st store \( \mu \), which is related to the C state using the state relation \( \mathcal{R} \), defined in \( \S 4.3 \).

Let \( \lambda x. M \)\( e \) and \( \lambda v. M \)\( r \)\( v \) (defined in \( \S 4.4 \)) be two functions that monomorphic expressions and (function) values respectively, using a rename function \( r \) provided by the compiler. Further, let \( s \) be a shallow embedding, \( e \) a monomorphic deep embedding, \( p\text{a} \) a C program, \( \mu \) a Coq'st store and \( s \) a C state. We define the overall refinement predicate \( \text{correspondence} \) as follows:

If \( (\exists w. U \mid \mu \vdash v : \Gamma : [r \cup w]) \) and \( (\mu, \sigma) \in \mathcal{R} \), then \( p\text{a} \) successfully terminates starting at \( \sigma \), and for any resulting value \( v\text{a} \) and state \( s' \) of \( p\text{a} \), there exist \( \mu' \), \( u \) and \( v \) such that:

\[
(\mu', s') \in \mathcal{R} \land U \vdash e \Downarrow v\text{a} \mid \mu' \land V \vdash e \Downarrow v, M \text{, } r \text{ v} \land \mathcal{V} \text{ r} \text{ } \mu' \text{ } v\text{a} \text{ u v s}
\]

Intuitively, our top-level theorem states that for related input values, all programs in the refinement chain evaluate to related output values. This can of course be used to deduce that there exist intermediate programs through which the C code and its shallow embedding are directly related. The user does not need to care what those intermediate programs are.

**Theorem 4.** Given a Coq'st function \( f \) that takes \( x \) of type \( \tau \) as input, let \( p\text{a} \) be its generated C code, \( s \) its shallow embedding, and \( e \) its deep embedding. Let \( v\text{a} \) be an argument of \( p\text{a} \), and \( u \) and \( v \) be the update and value semantics arguments, of appropriate type, for \( f \).

\[
\forall \mu \sigma. \forall v. v\text{a} \mid u \downarrow v \Rightarrow \text{correspondence } r \mathcal{R} (s \downarrow v) (M, e) (p\text{a} \mid v\text{a}) U \vdash \Gamma \mu \sigma
\]

where \( U = (x \mapsto u), V = (x \mapsto v), \) and \( \Gamma = (x \mapsto \tau) \).

This refinement theorem additionally assumes abstract functions in the program adhere to their specification and their behaviour stays unchanged when monomorphised.
4.2 Well-typedness

We first mention the well-typing theorems used in refinement. The Cogent compiler proves, via an automated Isabelle tactic, that the monomorphically deep embedding of the input program is well-typed. Specifically, the compiler defines funDef(f) in Isabelle and proves that each Cogent function f \( x = e \) matches the signature given by funDef\( (f) \). The typing proof for the polymorphic embedding is described in §4.4.

**Theorem 5** (Typing). Let \( f \) be a (monomorphic) Cogent function, where funDef\( (f) \) = \( \{ f : \tau \rightarrow \tau', f x = e \} \). Then \( \sigma \vdash x : \tau \triangleright e : \tau' \).

As we will see in §4.3, proving refinement requires access to the typing judgements for program sub-expressions and not just for the top level, so the Cogent compiler instructs Isabelle to store all intermediate typing judgements established during type checking. These theorems are stored in a tree structure, isomorphic to the type derivation tree for the Cogent program. Each node is a typing theorem for a program sub-expression.

4.3 C to Cogent Monomorphic Deep Embedding

This section outlines the bottom three (the red arrow, ① and ②) transformations in Fig. 1. Details of this proof automation can be found in Rizkallah et al. [2016]. First, the C code is converted to a SimpI embedding by the C parser used in the sel4 project [Klein et al. 2009]. This step is simple makes no effort to abstract from low-level C semantics.

The second step applies a modified version of AutoCorres to produce a monadic shallow embedding of the C code, and to prove that the SimpI embedding is a refinement of the monadic shallow embedding. We modify AutoCorres to make its output more predictable and to switch off its control-flow simplification and forcing it to output the shallow embedding in the nondeterministic state monad of Cock et al. [2008], where computation is represented by functions of type \( \text{state} \Rightarrow (\text{state} \times \text{result}) \). Here \( \text{state} \) is the global state of the C program, including global variables, while \( \alpha \) is the return-type of the computation. A computation takes input the global state and returns a set, \( \text{results} \), of pairs with new state and result value. The computation also returns a boolean, \( \text{failed} \), indicating the presence of undefined behaviour.

While AutoCorres was designed to ease manual reasoning about C, one use it as the foundation for automatically proving correspondence to the Cogent input program. One of the main benefits AutoCorres gives us is a typed memory model. Specifically, the state of the AutoCorres monadic representation contains a set of typed heaps, each of type 32 word \( \Rightarrow \alpha \), one for each type \( \alpha \) used on the heap by the C input program.

If the monadic code never fails, then the C code is type- and memory-safe, and free of undefined behavior [Greenaway et al. 2014]. We prove non-failure as a side-condition of refinement, basically using Cogent’s type system to guarantee C memory safety.

The ② step in Fig. 1 is a refinement proof between deeply-embedded Cogent and a shallow, monadic embedding of C. To phrase the refinement statement we first define how deeply embedded Cogent values and types relate to their corresponding values in the monadic embedding. This value-mapping is captured by a value relation val-rel\( _C \), generated in Isabelle automatically by the Cogent compiler, using ad hoc overloading. We must generate val-rel\( _C \) separately for each Cogent program because the types used in the shallow embedding depend on those used in the deep embedding. For example, C structs are represented directly as Isabelle records.

Given all the relation definitions for a particular Cogent program, the state relation \( R \) defines the correspondence between the store \( \mu \) which the Cogent update semantics manipulates, and the state \( \sigma \) for the monadic shallow embedding.

**Definition 1** (Monad-to-Update State Relation). \( (\mu, \sigma) \in R \) if and only if for all pointers \( p \) in the domain of \( \mu \), there exists a value \( v \) in the corresponding heap of \( \sigma \) at location \( p \), such that val-rel\( _C \mu(p) v \).

With \( R \) and val-rel\( _C \), we define refinement generally between a monadic computation \( p_m \) and a Cogent expression \( e \), evaluated under the update semantics. We denote the refinement predicate corre. Because \( \text{R} \) changes for each Cogent program, we parameterise corre by an arbitrary state relation \( R \). It is parameterised also by the typing context \( \Gamma \) and the environment \( U \), as well as by the initial update semantics store \( \mu \) and monadic shallow embedding state \( \sigma \).

**Definition 2** (Monad-to-Update Correspondence). \( \text{corre} R e p_m U \Gamma \mu \sigma = (\exists \alpha w_\mu w_\sigma, U \vdash e) \Rightarrow (\mu, \sigma) \in R \Rightarrow \langle \text{failed}(p_m\sigma) \land (v_{\alpha}, v_{\sigma}) \in \text{results}(p_m\sigma) \Rightarrow (\exists u_{\mu}, U \vdash e) \Rightarrow (\mu, \sigma) \in R \land \text{val-rel}_C \alpha u_{\mu} \rangle \).

The definition states that if the state relation \( R \) holds initially, then the monadic computation \( p_m \) cannot fail and, moreover, for all executions of \( p_m \) there must exist a corresponding execution under the update semantics of the expression \( e \) such that the final states are related by \( R \) and val-rel\( _C \) holds between their results. AutoCorres proves automatically that: \( \text{failed}(p_m\sigma) \Rightarrow \text{results}(p_m\sigma) \neq \emptyset \).

The refinement proof is automatic in Isabelle, driven by a set of syntax-directed corre rules, one for each Cogent construct. The proof procedure uses the fact that the Cogent terms are in A-normal form to reduce the number of cases that need considering and to simplify the higher-order unification problems that some of the proof rules pose to Isabelle.

Fig. 10. Two example corre rules
the call-graph of the input program. Currently, the tactic is limited to computing call graphs correctly only for programs containing up to second-order functions. We have not needed any higher orders in our applications, but the tactic can be extended using similar techniques if needed.

The final refinement theorem for this stage assumes that corre $e$ holds for all the abstract functions used in the program.

**Theorem 6.** Let $f$ be a (monomorphic) Coqgent function, such that $\text{funDef}(f) = (\lambda \tau. f \equiv e)$. Let $p_n$ be its monadic shallow embedding, derived from its generated C code. Let $u$ and $v_n$ be arguments of appropriate types for $f$ and $p_n$ respectively. Then:

$$\forall \mu. \text{val-rel}_{C} u v_n \longrightarrow \text{corres} R e (p_m v_n) (x \mapsto u) (x : \tau) \mu \sigma$$

### 4.4 Monomorphic to Polymorphic Deep Embedding

To complete the refinement step from the update to the value semantics, the compiler just applies **Theorem 3.** We establish the correctness of the compiler’s monomorphisation pass, moving upwards in Fig. 1 from a monomorphic to a polymorphic deep embedding in the value semantics.

The compiler generates a renaming function $r$ that, for a polymorphic function name $f_{\tau}$ and types $\tau$, yields the specialised monomorphic function name $f_{\mu}$. Just as we assume abstract functions are correctly implemented in C, we also assume that their behaviour remains consistent under $r$.

We write an Isabelle function to simulate the compiler monomorphisation phase, and prove that (1) the monomorphised program generated is identical to that produced by the compiler, and (2) the monomorphised program is a correct refinement of the polymorphic one. We define two Isabelle functions parameterised by $r$: $M_{\mu}$ for monomorphising expressions and $M_{\tau}$ for monomorphising (function) values.

Step (1) is proved by straightforward rewriting, and is automated on a per-program basis. Step (2) is embodied in the following refinement theorem, which we prove, once and for all, by rule induction over the value semantics. The specialisation Lemma 4 of §3.2.4 is a key ingredient of this proof.

**Theorem 7 (Monomorphism).** Let $f$ be a (polymorphic) Coqgent function whose definition given by $\text{funDef}(f) \equiv f \equiv e$. Let $r$ be an appropriately-typed argument for $f$. Let $r$ be a renaming function then:

$$\forall r'. (x \mapsto M_{\tau}, r v) \vdash M_{\tau} r e \equiv, M_{\mu} r v' \longrightarrow (x \mapsto v') \vdash e \equiv, v'$$

As the compiler generates a well-typedness proof for a monomorphic deeply embedded program (§4.2), **Theorem 7** implies well-typedness of the polymorphic deep embedding.

### 4.5 Deep to Shallow Embedding

In this section, the proof once again connects deep and shallow embeddings, where the shallow embedding is, this time, a pure function in Isabelle/HOL. This shallow embedding is still in A-normal form and is produced by the compiler: For each Coqgent type, the compiler generates a corresponding Isabelle/HOL type definition, and for each Coqgent function, a corresponding Isabelle/HOL constant definition. We drop the linear types and remain in Isabelle’s simple types, because we have already used the linear types to justify our switch to the value semantics.

In addition to these definitions, the compiler produces a theorem that the deeply embedded polymorphic Coqgent term under the value semantics correctly refines this Isabelle/HOL function. Refinement is formally defined here by the predicate $\text{corres}$ that defines when a shallowly embedded expression $s$ is refined by a deeply embedded one $e$ when evaluated under the environment $V$.

### 4.6 Shallow Embedding to Neat Shallow Embedding

Fig. 11 depicts the top-level shallow embedding, only mildly polished for presentation, for the Coqgent example of Fig. 2. As it shows, the Isabelle definitions use the same names as the Coqgent input program and keep the same structure, making it easy for the user to reason about.

The correctness statement for this phase is simple: it is pure Isabelle/HOL equality between the A-normal and neat shallow embedding for each function. For instance:

$$\text{Shallow.ext2_free_branch} = \text{Neat.ext2_free_branch}$$

The proof is simple as well. Since we can now use equational reasoning with Isabelle’s powerful rewritter, we just unfold both sides, apply extensionality and the proof is automatic given the right congruence rules and equality theorems for functions lower in the call graph.

### 5. Discussion

Our aim is to significantly reduce the cost of verifying real-world systems software with minimal performance impact. In this section, we evaluate the performance and usability of Coqgent in two realistic file system implementations, as well as discuss ways in which Coqgent can be improved in the future.

**Coqgent as a Systems Programming Language**

Running IO-Zone [10Z] microbenchmarks, we previously demonstrated that the ext2 implementation in Coqgent exhibits a modest improvement compared to the C version in random- and sequential-writes throughput, with similar CPU usage at around 10% [Amani et al. 2016]. For our own file system BilbyFs, the Coqgent version performs slightly worse than the C version, with 10–20% less throughput and CPU usage of 20% compared to 10%. In the absence of any I/O disturbance, the Coqgent implementation of ext2 is also slightly slower than native ext2fs when performing random writes to a RAM disk. These slight performance overheads are due to the fact that the Coqgent implementations tend to use data
structures that strongly resemble those of the original C, rather than more idiomatic functional data structures, which are handled better by our compiler. For some operations, the C implementation would rely on unsafe features for performance, where the Cogent implementation uses slower, but easier to verify techniques. One way to improve this on a language level would be to include specialised constructs for these operations in the language, allowing for the generation of efficient, yet automatically verified code. Including support for such constructs is at the top of the priority list for future work on Cogent. These case studies are invaluable for identifying common patterns and guide this important next step.

Occasionally, some overheads are introduced in C code generation, as we rely on the C compiler for low-level optimisation. Generated code displays patterns which are uncommon in handwritten code and therefore might not be picked up by the C optimiser, even if they are trivial to optimise. For example, the generated code is already quite close to SSA form used by gcc internally for optimisation, however gcc does not recognize this and optimise accordingly. Generating a compiler’s SSA representation directly, such as that of LLVM [LLVM], may eliminate these problems, however this would imply significant changes to our verification tool chain.

In addition to the results in Amani et al. [2016], we evaluated a set of standard applications on top of the C and Cogent BilbyFs implementations. The evaluation configuration is a Mirabox [Mirabox] with 1 GiB of NAND flash, a Marvell Armada 370 single-core 1.2GHz ARMv7 processor and 1 GiB of DDR3 memory, running Linux kernel 3.17 from the Debian 6a distribution. Fig. 12 shows the running time of the applications on top of native C BilbyFs compared to Cogent BilbyFs. The first scenario clones the git repository of the language Go [Go], creating 580 directory and 5122 files, for a total of 152MB. The next test compiles the Filebench-1.4.9.1 source code [Filebench Project] with 44 C files for a total of 19K SLOC. The third application extracts a large tarball (an archlinux distribution), and the fourth measures the total size of a large directory structure. The tar command has a 20% slowdown, but all other scenarios show the formally verified Cogent version performing within 5% of the unverified native C implementation.

User Experience We have found that the BilbyFs proofs were significantly easier and less time consuming than direct C proofs; partly because the purely functional shallow embedding enabled a high degree of automation with Isabelle/HOL, and partly because the linear type system provided free theorems: e.g. operations that do not write to disk represent the disk as a shareable type, which automatically establishes that no writes are performed to it. This illustrates that Cogent is practical and usable for writing real-world systems, that the generated C code is efficient, and that reasoning on top of the neat shallow embedding is much simpler than reasoning about C directly.

Effort and Size Cogent has been under development for over two years and has continually evolved as we have scaled it to ever larger applications. All up, the combined language development and certifying compiler took five person years. Engineering the Cogent compiler, excluding 33.5 person months (pm) spent on proof automation and proof framework development, consumed ten pm. The remaining 18 pm were for the design, formalisation and proof of Cogent and its properties (e.g. the theorems of §3). The total size of the development in Isabelle is 16,423 LOC, which includes the once-and-for-all language proofs plus automated proof tactics to perform the translation validation steps, given appropriate hints from the Cogent compiler. The Cogent compiler, written in Haskell, is 11,456 SLOC. For 9,447 lines of ext2 Cogent code the compiler generates 119,167 lines of Isabelle/HOL proofs and embeddings.

Optimisation The current Cogent compiler applies few optimisations when generating C code and leaves low-level optimisation to gcc or CompCert. Involved optimisations in the Cogent-to-C stage would complicate our current syntax-directed correspondence approach. Cogent-to-Cogent optimisations, however, are straightforward. The ease of proving A-normalisation correctness over the shallow embedding via rewriting suggests that this is the right approach in our context. In particular, some of the source-to-source optimisations discussed in Chlipala [2015] seem promising for Cogent.

6. Related Work

Like Cogent, HASP’s systems language [HASP project 2010], Habit, is a functional language. It has a verified garbage collector [McCreight et al. 2010], but no full formal language semantics yet. Ivory [Pike et al. 2014] is a domain specific systems language embedded in Haskell. It generates well-defined, memory safe C code, but unlike Cogent it does not prove its correctness.

Linear types have been used in several general purpose imperative languages such as Vault [Fahndrich and DeLine 2002] and Rust [Rust]. PacLang [Ennals et al. 2004] uses linear types to guide optimisation of packet processing on network processors. Similar substructural type systems, namely uniqueness types, have been integrated into the functional language Clean [Barendsen and Smeets 1993]. However, the intent is only to provide an abstraction over effects, and thus Clean still depends on run-time garbage collection.

Hofmann [2000] proves, in pen and paper, the equivalence of the functional and imperative interpretation of a language with a linear type system. The proof is from a first order functional language to its translation in C, without any pointers or heap allocation. In contrast, Coqsex is higher order, accommodates heap-allocated data, and its compiler produces a machine checked proof linking a purely functional shallow embedding to its C implementation.

As discussed in §1, Kumar et al. [2014] prove the correctness of the high-level language compiler CakeML. As it depends on runtime garbage collection, it is not suitable for our systems target. Furthermore, as it is an unrestricted, Turing-complete language with mutable state, its high-level semantics are not amenable to equational reasoning. By contrast, Neis et al. [2015] focus on a compositional approach to compiler verification for a relatively simple functional language, Pilsner, to an idealised assembly language. Like us, Charguéraud [2010, 2011] generates shallow embeddings to facilitate mechanical proofs, but unlike us they do not prove compilation correctness.

Chen et al. [2015] formally show in Coq [Bertot and Castérén 2004] full crash-resilience of FSCQ. FSCQ is smaller than ext2 and BilbyFs, and an order of magnitude slower than asynchronous ext4. Its implementation relies on generating Haskell code from Coq, and executing that code with a full Haskell runtime in userspace. We focus on bridging high-level specification and low-level implementation, on efficiency, and on providing a small trusted computing base, while Chen et al. [2015] assume all these are given and focus on crash resiliance. The approaches are complementary, i.e. it would be straightforward to implement Crash

<table>
<thead>
<tr>
<th>Application</th>
<th>C</th>
<th>Cogent</th>
</tr>
</thead>
<tbody>
<tr>
<td>git clone</td>
<td>84 sec</td>
<td>88 sec</td>
</tr>
<tr>
<td>golang</td>
<td>48 sec</td>
<td>50 sec</td>
</tr>
<tr>
<td>make filebench</td>
<td>80 sec</td>
<td>96 sec</td>
</tr>
<tr>
<td>tar</td>
<td>30 sec</td>
<td>31 sec</td>
</tr>
</tbody>
</table>

Figure 12: Running time for applications on top of BilbyFs
7. Conclusions
We presented a framework for dramatically reducing the cost of formal verification of important classes of systems code. It relies on the COGENT language, its certifying compiler, their formal definitions and top-level compiler certificate theorem, and the correctness theorems for each compiler stage. COGENT targets systems code where data sharing is minimal or can be abstracted, performance and small memory footprint are requirements, and formal verification is the aim.

COGENT is a pure, total functional language to enable productive equational reasoning in an interactive theorem prover. It is higher-order and polymorphic to increase conciseness. It uses linear types to make memory management bugs compile time errors, and to enable efficient destructive in-place update. It avoids garbage collection and a trusted runtime to reduce footprint. It supports a formally modeled FFI to interoperate with C code and to implement additional data types, iterators and operations. It does all of these with full formal proof of compilation correctness and type-safety in Isabelle/HOL.

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