## APPENDIX I

### NOTES1

## CHAPTER I

The calculus of matrices was first used in 1853 by Hamilton (1, p. 559ff, 480ff) under the name of "Linear and vector functions." Cayley used the term matrix in 1854, but merely for a scheme of coefficients, and not in connection with a calculus. In 1858 (2) he developed the basic notions of the algebra of matrices without recognizing the relation of his work to that of Hamilton; in some cases (e.g., the theory of the characteristic equation) Cayley gave merely a verification, whereas Hamilton had already used methods in three and four dimensions which extend immediately to any number of dimensions. The algebra of matrices was rediscovered by Laguerre (9) in 1867, and by Frobenius (18) in 1878.

- 1.03 Matric units seem to have been first used by B. Peirce (17); see also Grassmann (5, §381).
- 1.10 For the history of the notion of rank and nullity see Muir, *Theory of Determinants*, London 1906-1930; the most important paper is by Frobenius (290).

### CHAPTER II

- 2.01-03 The principle of substitution given in §2.01 was understood by most of the early writers, but was first clearly stated by Frobenius, who was also the first to use the division transformation freely (20, p. 203).
- 2.04 The remainder theorem is implicit in Hamilton's proof of the characteristic equation; see also Frobenius (280).
- 2.05-12 The characteristic equation was proved by general methods for n=3, 4 by Hamilton (1, p. 567; 8, p. 484ff; cf. also 4, 6). The first general statement was given by Cayley (2); the first general proof by Frobenius (18). See also the work of Frobenius cited below and 9, 10, 39, 41, 56, 59.

Hamilton, Cayley and other writers were aware that a matrix might satisfy an equation of lower degree than n, but the theory of the reduced equation seems to be due entirely to Frobenius (18, 140).

The theory of invariant vectors was foreshadowed by Hamilton, but the general case was first handled by Grassmann (5).

- 2.10 See Sylvester (42, 44) and Taber (96); see also 252.
- 2.13 The square root of a matrix was considered by Cayley (3, 12), Frobenius (139) and many others.

# CHAPTER III

3.01 The idea of an elementary transformation seems to be due in the main to Grassmann (5).

<sup>&</sup>lt;sup>1</sup> In these Notes, numbers refer to the Bibliography unless otherwise indicated.

3.02-07 The theory of pairs of bilinear forms, which is equivalent to that of linear polynomials, was first given in satisfactory form by Weierstrass (see Muth, 175) although the importance of some of the invariants had been previously recognized by Sylvester. The theory in its matrix form is principally due to Frobenius (18, 20).

The theory of matrices with integral elements was first investigated by Smith (see Muth, 175) but was first given in satisfactory form by Frobenius (20). The form given in the text is essentially that of Kronecker (92).

- 3.04 The proof of Theorem 3 is a slight modification of that of Frobenius (20).
- 3.08 Invariant vectors were discussed by Hamilton (1, 8) and other writers on quaternions and vector analysis. The earliest satisfactory account seems to be that of Grassmann (5).

## CHAPTER IV

The developments of this chapter are, in the main, a translation of Kronecker's work (see Muth, 175, p. 93ff). See also de Séguier (259).

#### CHAPTER V

- 5.03 From the point of view of matrix theory, the principal references are Schur (198), Rados (105, 106), Stephanos (185), and Hurwitz (117). See Loewy (284, p. 138) for additional references; also Muir, *Theory of Determinants*, London 1906-1930.
- 5.09 Non-commutative determinants were first considered by Cayley (Phil. Mag. 26 (1845), 141-145); see also Joly (195) and Sylvester (43).
- 5.10-11 See Loewy (284, p. 149); also 176, 178, 185, 198.
- 5.12 The principal references are Schur (198) and Weyl (440, chap. 5).

## CHAPTER VI

For general references see Loewy (284, pp. 118-137), also Muth (175), Hilton (314, chap. 6, 8) and Muir, Theory of Determinants, London 1906-1930.

- 6.01 The method of proving that the roots are real is essentially that of Tait (10, chap. 5); see also 36, 60, 228, 399.
- 6.03 See Loewy (284, pp. 130-137), Baker (215) and Frobenius (292). See also 7, 18, 99, 113, 114, 115, 124, 135, 139, 210, 221, 273, 302, 307, 320, 371, 400, 414, 466, 476.
- 6.04 See Dickson (392).
- 6.05 See Loewy (284, pp. 128-135).
- 6.07 For references see Muth (175, p. 125) and Frobenius (139).

#### CHAPTER VII

- 7.10-02 See Cayley (2), Frobenius (18), Bucheim (59), Taber (98, 112), and Hilton (314, chap. 5); also 83, 86, 98, 137, 184, 197, 209, 223, 242, 250, 264, 301, 382.
- 7.03 See Frobenius (280).
- 7.05 See Frobenius (140); also 350.
- 7.06-07 See Sylvester (42, 44) and Taber (96); see also 252.

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## CHAPTER VIII

- 8.01-03 See Sylvester (36), Bucheim (59, 69); also 134, 371.
- 8.02,07 See Hamilton (1, p. 545ff; 8, §316), Grassmann (5, §454), Laguerre (9). Many writers define the exponential and trigonometric functions and consider the question of convergence, e.g., 79, 80, 103, 389, 449; also in connection with differential equations, 13, 133, 258.
- 8.04-05 Roots of 0 and 1 have been considered by a large number of writers; see particularly the suite of papers by Sylvester in 1882-84; also 18, 67, 76, 107, 242, 255, 264, 277, 279, 381, 411, 430, 474, 539.
- 8.08 See 20, 94, 246, 256, 257, 274, 303, 338, 399.
- 8.09-11 The absolute value of a matrix was first considered by Peano (75) in a somewhat different form from that given here; see also 273, 348, 389, 472, 473, 494. For infinite products see 133, 324, 326, 389, 494.
- 8.12 In addition to the references already given above, see 10, 16, 18, 187, 418, 419, and also many writers on differential equations.

## CHAPTER IX

The problem of the automorphic transformation in matrices was first considered by Cayley (3, 7) who, following a method used by Hermite, gave the solution for symmetric and skew matrices; his solution was put in simpler form by Frobenius (18). Cayley failed to impose necessary conditions in the general case which was first solved by Voss (85, 108, 162, 163). The properties of the principal elements were given by Taber (125, 134; see also 127, 149, 156, 158, 231). Other references will be found in Loewy (284, pp. 130-137); see also 9, 19, 153, 154, 161, 167, 168, 169, 187, 229, 371.

### APPENDIX II

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