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BORIS NIKOLAEVICH DELONE (ON HIS LIFE AND CREATIVE WORK)

UDC 51(092) Delone

Boris Nikolaevich Delone was, first and foremost, a great mathematician, and mathematical creativity was certainly an essential part of the life of this remarkable scholar. But to speak of Boris Nikolaevich only as a mathematician is impossible, just as it is impossible to isolate in a complex polyphonic composition its main theme, however striking it may be, so inseparably is it linked to the other themes. The life of B. N. Delone—and he lived to the age of 90—was exceptionally full and multifaceted. Even a simple listing of the most distinctive aspects of his life would require more than one article.

In such a list one of the first entries, chronologically speaking, would be the year 1897, when the seven-year-old boy paints his first landscapes in oils, reads Goethe's *Faust* in the original, and memorizes various scenes of that tragedy.

Somewhere at the end of the list would be the date July 6, 1975, when Boris Nikolaevich, at the age of 86, spends the night in $-25°C$ weather on a glacier below the summit of the 7,000-meter high Khan Tengri(1) at a height of 4200 meters. In the morning he descends by helicopter to Lake Issyk-Kul and then flies to Frunze(2), where the temperature is $40°C$. Towards evening he turns up at an airport near Moscow and is then faced with the prospect of having to travel to his cottage in the Abramtsevo suburbs. Arriving at the station on the last train, Boris Nikolaevich walks through the woods late at night with a heavy rucksack and arrives home before dawn.

These biographical details show that Delone's many talents manifested themselves at a very early age and that in his advanced years he was able to maintain a lively youthful spirit, and his uncommon physical health enabled him to devote himself with complete efficiency, even near the end of his life, both to scientific work and to hiking expeditions. The geometry of numbers, mathematical crystallography, and discrete geometry were the subjects of papers written by Boris Nikolaevich when he was over 80. In his old age he made trips to the Altai, Caucasus, Carpathian, and Tien Shan mountains.

Boris Nikolaevich Delone was born in 1890, the son of Nikolaĭ Borisovich Delone, a professor of mechanics. He received, as remarked previously, an excellent education. He took a serious interest in music. He knew well and loved the works of Bach and Mozart. He played all of Beethoven's sonatas and composed many of his own. His music teacher urged the gifted youth to enroll in a composition class in a conservatory. But no less justified was his drawing teacher in recommending that

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(1) *Translator's note.* At 23,620 feet, or 7200 meters, the highest peak of the central Tien Shan range in Central Asia.

(2) *Translator's note.* This was the capital of the Republic of Kirgizstan.

Boris Delone continue his education in the Academy of Art. His occupation with drawing and painting revealed his talent and serious attitude.

While his teachers and parents pondered his future, the boy painted landscapes and played soccer, reproduced in pencil Leonardo's *Last Supper* and climbed trees (with his younger sister on his shoulders), composed musical pieces and traveled to the mountains. He turned his room into a physics laboratory and made many of the instruments himself. Later he often recalled with a sense of pride his little laboratory tricks, enabling him, in particular, to obtain "su-u-ch a nice spark" (the words accompanied by an inimitable expressive gesture) by means of a Leyden jar.

Having taken an interest in astronomy, he constructed a telescope and polished the mirror for it himself. When speaking of this, Boris Nikolaevich never forgot to add: "Polishing a bronze mirror was foolish, because it was laborious and the mirror quickly lost its luster."

Boris Nikolaevich's father, Nikolaĭ Borisovich, was a close acquaintance of the "father of Russian aviation", Nikolaĭ Egorovich Zhukovskiĭ. Under Zhukovskiĭ's influence, Nikolaĭ Borisovich organized Russian's first aeronautical society in Kiev in 1907. The 17-year-old Boris Nikolaevich actively participated in the work of the society and, during the next two years, built five gliders, gradually improving their construction, and flew them, sometimes at the risk of his life. (Once, some cinematographers who had come to shoot a film on the work of the society persuaded him, despite the presence of a strong wind, to take to the air. A gust of wind overturned the glider, and Boris fell 15 meters to the ground, fortunately into a deeply plowed field.)

Of course, from the modern point of view the achievements of the society may seem naive: flights of several dozen meters and launches by means of a rope and a horse, and using a sleigh in winter and a bicycle in summer. But this was very early in the history of aviation, and the members of the society were really pioneers. The work of the Kiev society did influence the development of a native aircraft industry. Let it suffice to say that one of the members of the society was Igor′ Ivanovich Sikorskiĭ. It is interesting that Andreĭ Nikolaevich Tupolev says in his memoirs, when writing of the considerable significance of his acquaintance with Zhukovskii, that he was introduced, while still a young student, to Nikolaĭ Egorovich by B. N. Delone.

The mathematical talent of Boris Nikolaevich became apparent rather early. By the age of 12 he knew the fundamentals of analysis, and a little later began independent investigations in algebra and number theory. His familial surroundings were undoubtedly conducive to the development of this aspect of his talent. His father, for example, took him to the International Congress in Heidelberg, where the 14-year-old Delone saw and heard Hilbert and Minkowski. He was often present at conversations between his father and the eminent Russian mathematician Georgiĭ Feodos′evich Voronoĭ. Later on, Voronoĭ's papers, and perhaps these conversations, had an enormous influence on Delone's mathematical creativity. However, there were no personal scientific contacts between Boris Nikolaevich and Voronoĭ. Voronoĭ died at the age of 40 in 1908, the same year that Boris Nikolaevich entered the Physics and Mathematics Department of Kiev University.

At Kiev University, B. N. Delone, O. J. Schmidt, who entered at the same time, and N. G. Chebotarev, who came a year later, took part in the algebra seminar of Professor D. A. Grave, which became famous as a result. This seminar determined, for a long time, the range of Delone's research interests: algebra and number theory. His 1912 paper entitled *The connection between the theory of ideals and Galois theory*, a subject suggested by the faculty, was awarded the university's Large Gold Medal.

His first publication, *On the definition of an algebraic domain by means of congru-*

ences, appeared in 1915. In it Delone gave a short proof of the Kronecker-Weber theorem on absolutely Abelian fields. This paper, along with others of Boris Nikolaevich from the period 1912–1916, considerably influenced the research interests of N. G. Chebotarev, who in his autobiography calls himself basically a student of B. N. Delone.

One of the high points of Delone's mathematical creativity is a series of investigations in the theory of third-degree Diophantine equations in two unknowns. Let us go into this in more detail.

In various important questions in number theory that have stimulated its development since ancient times there appears the problem of investigating Diophantine equations of the form

$$f(x_1, \dots, x_n) = 0,$$

where f is a polynomial with integral coefficients and it is required that the solutions consist of integers. Despite the extreme simplicity of the formulation of the problem, the theory of Diophantine equations is not rich in general results. This situation was, apparently, one of the reasons for the appearance of Hilbert's tenth problem. As Boris Nikolaevich recalled, it was "pe-erfectly clear" to all specialists in Diophantine equations that this problem must have a negative solution, i.e. there could be no single algorithm for solving Diophantine equations. (It was only relatively recently that the profundity of the intuition of these specialists was confirmed by results of Yu. V. Matiyasevich.)

As for concrete investigations, even the simplest second-degree indeterminate equation, the so-called Pell equation

$$x^2 - ay^2 = 1,$$

first studied by Euler, has a completely nontrivial, beautiful solution, connected with the periodicity of the expansion of $\sqrt{a}$ into a continued fraction. An investigation of the general second-degree Diophantine equation was carried out by Lagrange.

As for solutions of Diophantine equations of higher degree, there were no major successes in this area right up to the start of the twentieth century. This endowed the question of solving third-degree Diophantine equations with the status of a difficult problem. It was this problem that Boris Nikolaevich, who was always inspired to undertake difficult, important tasks and not waste his time and talent on trifles, chose.

Delone's investigations were focused on Diophantine equations

$$f(x, y) = 1,$$

where $f(x, y)$ is a third-degree form with negative discriminant.

He began with the cubic analogue of Pell's equation,

$$ax^3 + y^3 = 1, \tag{1}$$

where the integer a is not a perfect cube. (If a is a perfect cube, the cubic Pell equation reduces to first- and second-degree Diophantine equations.)

The question is: Does the cubic Pell equation have solutions other than the trivial solution $(0, 1)$, and what are they?

The answer to this question is connected with the study of the ring σ of algebraic numbers of the form

$$A\sqrt[3]{a^2} + B\sqrt[3]{a} + C \qquad (A, B, C \in \mathbf{Z}),$$

where a is the coefficient in the Pell equation.

If (X_0, Y_0) is a solution of (1), then the number $\varepsilon = X_0\sqrt[3]{a} + Y_0$ belongs to the ring σ and so does ε^{-1}, i.e. ε is a unit of this ring and has a particular form: it is a two-term unit with the coefficient of $\sqrt[3]{a^2}$ equal to zero. It follows from Dirichlet's theorem on units that $\varepsilon = \varepsilon_0^m$, where ε_0 is a fundamental unit of σ and m is an integer, and if ε_0 is chosen so that $0 < \varepsilon_0 < 1$, then $m > 0$.

Then Delone showed, by an ingenious arithmetical argument, that a two-term unit of the indicated type (with coefficient of $\sqrt[3]{a^2}$ equal to zero) must be a fundamental unit. Thus he proved:

The equation $ax^3 + y^3 = 1$, where a is an integer not a perfect cube, can have at most one solution in integers beyond the trivial solution $(0, 1)$; a nontrivial solution exists if and only if the fundamental unit ε_0 has two terms and has the form $\varepsilon_0 = X_0\sqrt[3]{a} + Y_0$, in which case the nontrivial solution is (X_0, Y_0).

Twenty years earlier, Voronoĭ had given an algorithm for calculating this fundamental unit in a ring lying in a third-degree field with negative discriminant. For example, the fundamental unit for the equation $4x^3 + y^3 = 1$ has three terms, hence this equation has no solutions other than $(0, 1)$. However, the equation $37x^3 + y^3 = 1$, which has fundamental unit $\varepsilon_0 = -3\sqrt[3]{37} + 10$, has two solutions, $(0, 1)$ and $(-3, 10)$.

After an exhaustive investigation of the cubic Pell equation, Delone turned to the general case, i.e. the equation

$$f(x, y) = 1, \tag{2}$$

where $f(x, y) = x^3 + ax^2y + bxy^2 + cy^3$ is a cubic form with negative discriminant and integral coefficients a, b, and c.

Using his "algorithm of ascent", Delone obtained the following key result: The general equation has at most five solutions in integers. Moreover, in each particular case the algorithm actually provides the means of finding the solutions, if they exist.

Boris Nikolaevich also showed, by means of the algorithm of ascent, that there are only two classes of integrally equivalent equations of type (2) having exactly four solutions, and only one class of equations, namely the equivalence class of the equation $x^3 - xy^2 + y^3 = 1$, has five solutions (the solutions of this equation are $(1, 1)$, $(1, 0)$, $(0, 1)$, $(-1, 1)$, and $(4, -3)$). Any other equation of type (2) has at most three solutions.

Delone's work on cubic Diophantine equations represented almost the first substantial progress since Euler and Lagrange in the concrete investigation of equations of higher degree. Recall that other remarkable results on indeterminate equations had been obtained by Axel Thue at the beginning of this century. The intersection of his results with those of Boris Nikolaevich revealed only the finiteness of the number of solutions of equations of type (2).

We can say that in some sense Delone's results remained unsurpassed for about half a century, to the end of the 1960s. And for concreteness, simplicity, and clarity, Delone's papers on indeterminate equations are exceptional in twentieth century mathematics, with its often cumbersome machinery and abstract constructions. The style of these papers approaches the high standards set by the classic papers of Gauss and Chebyshev on number theory. Boris Nikolaevich himself, in summing up his accomplishments, frequently said that his papers on indeterminate equations undoubtedly represented his most difficult and elegant work.

After completing his papers on Diophantine equations, Delone still had in front of him 60 years of creative activity, which added to his reputation as a great algebraist that of a great geometer. The shifting of his investigations towards geometry occurred naturally. A definite factor here was his contact, during his investigations on indeterminate equations, with the work of Voronoĭ. But the main reason, in our opinion, for the geometrization of Delone's investigations was his exceptionally graphic, artistic perception of the world. It is characteristic that Boris Nikolaevich never regretted the time and effort required to bring the comprehension of a seemingly already complete mathematical result to the point where it could be given a visible, geometric shape. "What does this mean in simple terms?" was a favorite question of Boris Nikolaevich, which he posed both to himself and his colleagues when discussing the paper at hand.

Along with his investigations of indeterminate equations, and in connection with them, Boris Nikolaevich considered a number of other questions related to the theory of cubic irrationalities, and characteristic of this period there was, as we have already mentioned, an obvious geometrization of his investigations. For example, in 1923 Boris Nikolaevich gave a brilliant geometric interpretation of Voronoĭ's above-mentioned algorithm for finding a fundamental unit. Then Boris Nikolaevich constructed a beautiful geometric theory of binary cubic forms and its covariants. In these papers he made use of geometry, interpreting a ring of cubic irrationalities as a three-dimensional lattice with a certain natural multiplication.

Later on, in the 1930s, he again returned to developing the geometric point of view in the theory of algebraic number fields and "discrete" rings in these fields. His results in the theory of indeterminate equations and the theory of cubic irrationalities and ideas of his that were developed by his students were expounded in a well-known monograph written jointly with D. K. Faddeev, *The theory of irrationalities of the third degree* (1940; English transl., 1964). A further development of the geometric approach to solving equations by radicals led Boris Nikolaevich to a geometric exposition of Galois theory. His papers introduced visualization into this very abstract mathematical theory.

As for the second half of the 1920s, Boris Nikolaevich, apparently influenced by papers of G. F. Voronoĭ, E. S. Fedorov, and Hermann Minkowski, worked on purely geometric problems.

In one important paper (1929), Boris Nikolaevich determined all four-dimensional parallelohedra, both primitive and imprimitive, by studying the so-called closed zones of four-dimensional parallelohedra. The most essential and difficult feature of that paper is Delone's theorem that for each four-dimensional parallelohedron a closed zone always exists. The presence of such zones enabled him (by studying "caps" of parallelohedra) to reduce to some extent the question to the three-dimensional case, which led to a complete solution.

That paper, like the ones on indeterminate equations, is a model of concreteness and remains the definitive work on the subject.

Much later, in the late 1950s and early 1960s, Boris Nikolaevich returned to the problem of dividing a space into polyhedra (we should say that returning to old subjects after having acquired more experience was very characteristic of Boris Nikolaevich). At the end of the 1950s Boris Nikolaevich began to study regular partitions of n-space with an arbitrary Fedorov group. He began his investigation with an important paper in which the plane case was treated in exhaustive detail. Turning to the general n-dimensional case, Delone proved in 1961 the following fundamental theorem on stereohedra: The number of combinatorial-topological types of partitions of Euclidean n-space into convex normal (i.e. contiguous along entire $(n-1)$-faces) stereohedra is finite. In proving this theorem, Boris Nikolaevich gave an upper bound for the number of faces of a stereohedron, so that Minkowski's famous estimate of the number of faces of a parallelohedron became a special case of Delone's estimate.

On the basis of that paper, Boris Nikolaevich and N. N. Sandakova jointly solved in principle the problem of determining, for any given n, all combinatorial-topological types of regular Dirichlet partitions of Euclidean n-space.

Yet another topic of study, which Boris Nikolaevich began to pursue in the 1920s and never abandoned, was geometric crystallography. The formulation of the basic problems of this beautiful science, which has important practical applications, is similar to the formulation of the basic problems of the theory of positive ternary forms. This is due to the fact that among the basic objects of geometric crystallography are three-dimensional lattices.

Boris Nikolaevich interpreted and systematized the foundations of geometric crytallography in his own way. Having established a connection between Voronoï-Dirichlet domains and Selling reduction of ternary forms, Boris Nikolaevich discovered in 1933 an algorithm for solving the problem concerning the regular arrangement of a crystal. Delone's method, which involved the use of his tetrahedral symbol, served until very recently as the basis for describing a crystal in international tables.

A fundamental discovery of Delone was the ascertainment of 24 kinds of lattices depending on the combinatorial structure of a Voronoï-Dirichlet domain and the relative distribution of the elements of symmetry.

The crystallography papers of Boris Nikolaevich that stem from his first period of involvement in this area were made into the monograph *Mathematical foundations of the structural analysis of a crystal* (1934), which he wrote in conjunction with A. D. Aleksandrov and N. N. Padurov. For the next 50 years geometric crystallography continually aroused the interest of Boris Nikolaevich and was, in fact, the subject of his last scientific paper.

In the mid-1960s, Boris Nikolaevich and his students investigated two extremal problems of lattice theory. First they considered the problem of the least dense lattice coverings of n-space by equal spheres. In particular, Boris Nikolaevich and S. S. Ryshkov jointly solved the problem for $n = 4$ (for $n = 3$ this problem was solved in 1954 by the well-known Indian mathematician R. P. Bambah). The other extremal problem that drew the attention of Boris Nikolaevich and his students was that of n-dimensional lattices with given volume of the fundamental parallelepiped minimizing $\sum r^{-2m}$, where r ranges over the distances to all points of the lattice from one of them and $m > n/2$. They also obtained important results on this problem. These problems are not only of theoretical, but also of practical, interest, in particular with regard to S. L. Sobolev's well-known results on cubature formulas.

It is noteworthy that most of Boris Nikolaevich's geometric investigations are connected, in one way or another, with the so-called empty sphere method, which he presented in 1924 at the International Mathematical Congress in Toronto. Boris Nikolaevich "propelled" an empty sphere between points of an arbitrary, uniformly discrete point system, increasing the radius of the sphere for as long as possible. For each limiting position of the empty sphere he considered the convex hull of the points lying on its boundary. It turns out that the resulting polyhedra fill up the whole space and are contiguous along entire faces. Boris Nikolaevich showed that if the point system is a lattice, this partition agrees with the L-partition defined by Voronoï. The vertices of the Voronoï-Dirichlet domains coincide with the centers of the empty spheres, and the Voronoï-Dirichlet partition itself is dual to the L-partition. Today L-partitions play an important role in computer geometry and, because of C. Rogers, are called Delone partitions (triangulations).

Boris Nikolaevich frequently told of an amusing incident in the history of the empty sphere method. When Delone first noticed in one of H. S. M. Coxeter's papers some constructions involving an empty sphere, he wrote Coxeter a letter saying that the empty sphere method had been announced in 1924 in the very same Toronto where Coxeter lived and worked. Coxeter's answer, according to Boris Nikolaevich, was, "In 1924 I was still in short pants and could not attend your talk."

Recognition of his scientific achievements was not long in coming: in 1929 B. N. Delone was made a Corresponding Member of the Academy of Sciences of the USSR.

B. N. Delone's brilliant scientific creativity meshed very well with his remarkable pedagogic activity. In 1923 he was made a professor at Leningrad University. His extremely clear, carefully planned lectures greatly inspired his students. He never missed an opportunity to tell them of a difficult unsolved problem. In his pedagogic practice he was guided by the well-known principle: "A student is not a vessel to be filled up, but a torch that must be ignited." His lofty scientific reputation, his devotion and aesthetic approach to science, his personal charm, and his artistic nature all attracted young people to him. It is not surprising that Boris Nikolaevich had many students, several of whom became outstanding mathematicians: Academician A. D. Aleksandrov and Corresponding Members D. K. Faddeev and I. R. Shafarevich.

In the spring of 1934, in Leningrad, B. N. Delone organized the first Mathematical Olympiad for high school students, the beginning of the Olypiad movement in our country. The participants in the Leningrad Olympiad still remember the "stunning impression" the meetings with Boris Nikolaevich made on them.

Boris Nikolaevich also contributed enormously to the organization of the Moscow Mathematical Olympiad and scholastic mathematics clubs at Moscow State University. One of the authors of this article had occasion to attend, at the end of the 1940s, some of Boris Nikolaevich's lectures for high school students in which he proved Zhukovskiĭ's theorem on the carrying capacity of the wing of an airplane (this event largely determined the destiny of said author).

Boris Nikolaevich's educational propensity was in no way limited to scholastic matters. In the journal *Priroda* there appeared, from time to time, Boris Nikolaevich's splendid short essays such as *Bolyai and Lobachevsky* and *Max Laue?*, and many others. With regard to one of these articles, an essay on the theorem of Zhukovskiĭ just mentioned, Boris Nikolaevich liked to say, "Our eminent academician, Pavel Sergeevich Aleksandrov, never flies in an airplane. And do you know why? He does not know the proof of Zhukovskiĭ's theorem and so does not understand how an airplane is supported in the air. But I know! And I am not afraid to fly!" Indeed, Boris Nikolaevich did not fear machinery and in fact, as we have already noted, was very fond of it. It is no wonder that among his educational and pedagogic activities we find the first course at Moscow State University on mechanical calculators.

At the founding of the V. A. Steklov Mathematical Institute B. N. Delone chaired the Algebra Department (until 1960) and then the Geometry Department. In 1935, when the Mathematical Institute and other important parts of the Academy of Sciences of the USSR were moved to Moscow, Boris Nikolaevich became chairman of the Geometry Department at Moscow State University. During his tenure at Moscow State University he gave an innovative course on analytic geometry noted for its extraordinary wealth of geometric ideas. It is unlikely that the students who had the good fortune to hear Delone's lectures suspected the painstaking preparation concealed by their elegance. He polished each idea, adjusted each word, perfected each figure. As a result, during his lectures there appeared effortlessly and quickly on the blackboard likeable "affine cats", graceful paraboloids and hyperboloids, and accurate depictions of dodecahedra and icosahedra. At one time there was in vogue among the students of mechanics and mathematics at Moscow State University a series of jokes about their professors on the subject, "Where does [the professor] make soup?" According to the student version, not devoid of irony, Boris Nikolaevich made soup in an n-dimensional lattice—the soup leaked out, to be sure, but the geometric visualization remained.

There were many legends about Delone. Once in a lecture, in order to erase the upper part of the blackboard, he began to jump up and down, after which he said to the students with his inimitable intonation, "You know that I am not only a famous mathematician, but a well-known mountain climber. You, most likely, not only don't know mathematics, but can't even do a handstand." And with that he did a handstand on the table. At the time he was in his seventies.

Boris Nikolaevich was indeed a well-known mountain climber. His enthusiasm for mountaineering was engendered in his early youth, when he climbed Mont Blanc, Monte Rosa, and other Alpine peaks. Later he made many climbs in the Caucasus and Altai Mountains. In 1931 Boris Nikolaevich organized in the Caucasus, for the workers of the Leningrad factory Krasnyi Putilovets, the first mountaineering camp in the country. In his book *Peaks of the Western Caucasus* he described several dozen principal peaks of that region. The description was accompanied by drawings, which he did himself, of each peak and a panorama of the region as a whole. B. N. Delone developed principles, functional even today, for the technical classification of peaks. In this classification, the peaks and routes of ascent are divided into ten categories according to their degree of difficulty. In 1935 B. N. Delone was awarded the honorary title *Master of Soviet Mountaineering*. One of the peaks of the Akkem wall of Belukha(3) is named in his honor—Delone's Peak.

With advancing age, his mountain climbing was replaced by hiking trips. Each summer he traveled to the mountains and made an arduous hike to the Shavlinsky lakes, or to the wall of Belukha in the Altai range, or to the Inylchek glacier in the heart of the Tien Shan range, or to Dombai, or elsewhere. As for the rest of the year, each week without fail, regardless of the weather, Boris Nikolaevich made 30–40-kilometer trips on foot or skis to the most beautiful places near Moscow. Boris Nikolaevich was exceptionally fit physically. Nevertheless it was uncomfortable to observe at winter's end this elderly man, stripped to the waist, walking briskly along with a rucksack on his back. The falling snow melted and, trickling down his gray head, formed large icicles.

Boris Nikolaevich loved nature and always contemplated its beauty in silence. His communion with nature was like a sacrament, and he found it unpleasant when this sacrament was inadvertently interrupted by conversation. His frame of mind was instinctively transmitted to his companions. But around a campfire or on a train they were treated to many stories, each more interesting than the one before. Boris Nikolaevich was a wonderful raconteur. "You, of course, do not know that my cousin is a saint," and then he paused, glancing slyly at his puzzled audience. "Yes, yes, I'm quite serious," and thus began a most interesting story about his first cousin E. Kuz'mina-Karavaeva, a Russian patriot, nun, and heroine of the French Resistance (she was the subject of the film *Mother Mary*).

In a salutory address on the occasion of his 80th birthday, the Leopoldina wished to its member Boris Nikolaevich Delone, Corresponding Member of the Academy of Sciences of the USSR, a "tranquil evening of life". This "evening of life" lasted another ten years and did not turn out to be very tranquil—it proved worthy of his entire life.

But his life neared its natural limit. We will never forget how, three months before the end, Boris Nikolaevich read a wonderful passage from Poincaré. His

(3) *Translator's note.* Belukha (White Mountain) is the highest peak (14,783 feet) in the Altai Mountains.

voice trembled, "... life is only a cursory episode between eternities of death and ... in this episode the past and future duration of a conscious thought is no more than an instant. A thought is only a flash of light in the middle of a long night. But this flash is everything... "

D. K. Faddeev, N. P. Dolbilin,
S. S. Ryshkov, and M. I. Shtogrin

Translated by G. A. KANDALL