CIS 5150
Fundamentals of Linear Algebra and Optimization
Syllabus

(1) Vector Spaces, Bases, Linear Maps

   (a) Groups, Rings, Fields and Vector Spaces
   (b) Indexed Families
   (c) Linear Combinations and Linear Independence
   (d) Linear Subspaces
   (e) Bases, the Replacement Lemma
   (f) Matrices
   (g) Linear Maps, Kernels, Images, Isomorphisms

(2) Matrices and Linear Maps

   (a) Matrix Representation of Linear Maps
   (b) Change of Basis Matrix
   (c) Direct Products and Direct Sums
   (d) Multiplication by Blocks
   (e) The Rank–Nullity Theorem and Applications
   (f) Grassmann's Relation
   (g) Affine Maps

(3) Haar Bases and Haar Wavelets

   (a) Signal Compression
   (b) Haar Matrices, Haar Wavelets
   (c) Digital Signal Resolution

(4) Determinants

   (a) Alternating Multilinear Maps and Determinants
   (b) Calculating Determinants via Laplace Expansion
   (c) Invertibility and Linear Independence
   (d) The Cayley–Hamilton Theorem

(5) Vector Norms and Matrix Norms

   (a) Normed Vector Spaces
   (b) Matrix Norms
   (c) Subordinate Norms
   (d) Condition Number

(6) Solving Systems of Linear Equations

   (a) Gaussian Elimination
   (b) LU-Factorization
   (c) LU-PA-Factorization
   (d) SPD Matrices and Cholesky Decomposition
   (e) Reduced Row Echelon Form (RREF)

(7) The Dual Space and Duality

   (a) The Dual Space E^* and Linear Forms
   (b) Pairing and Duality Between E and E^*
   (c) The Bidual and Canonical Pairing
   (d) Hyperplanes and Linear Forms
   (e) Transpose of a Linear Map and of a Matrix
   (f) The Four Fundamental Spaces

(8) Euclidean and Hermitian Spaces

   (a) Inner Products, Cauchy-Schwarz Inequality
   (b) Orthogonality and Duality
   (c) Adjoint of a Linear Map
   (d) Construction of Orthonormal Bases; Gram–Schmidt Orthonormalization Procedure
   (e) Linear Isometries and the Orthogonal Group
   (f) Orthogonal Matrices and QR-Decomposition
   (g) Hermitian Spaces and Unitary Transformations

(9) Eigenvalues, Eigenvectors and the Spectral Theorems

   (a) Eigenvalues and Eigenvectors
   (b) Triangular Form (Schur Form)

(c) Location of Eigenvalues (The Gershgorin Discs)
(d) Normal Linear Maps
(e) Spectral Theorem for Normal Linear Maps
(f) Spectral Theorem for Self-Adjoint, Skew-Adjoint and Orthogonal Linear Maps
(g) Matrix Versions of the Spectral Theorems
(h) Rayleigh Ratio, Rayley–Ritz Theorem

(10) Singular Value Decomposition

(a) Properties of $A^\top A$ and $AA^\top$
(b) SVD for Square Matrices
(c) Polar Form
(d) SVD for Rectangular Matrices

(11) Applications of SVD

(a) Least Squares and the Pseudo-Inverse
(b) Properties of the Pseudo-Inverse
(c) Data compression; Best Rank k Approximation.
(d) Principal Component Analysis (PCA)
(e) Best Affine Approximation

(12) Derivatives

(a) Open Sets and Continuity in Metric Spaces and Normed Spaces
(b) Limits and Continuity; Uniform Continuity
(c) Continuous Linear Maps and Continuous Multilinear Maps
(d) Directional Derivatives and Total Derivatives
(e) Jacobian Matrices
(f) Lagrange Multiplier Technique

(13) Quadratic Optimization Problems

(a) Unconstrained Positive Definite Optimization
(b) Constrained Positive Definite Optimization
(c) General Quadratic Optimization
(d) Maximizing a Quadratic Function on the Unit Sphere

(14) Introduction to Nonlinear Optimization

(a) Optimization Terminology
(b) Convex Sets and Convex Functions
(c) Active Constraints, Qualified Constaints
(d) KKT Conditions
(e) Duality
(f) Equality Constraints
(g) Hard Margin SVM (Support Vector Machine)

(15) Soft Margin Support Vector Machine

(a) Formulation of ν-Soft Margin SVM as an Optimization Problem
(b) Classification of Data Points
(c) Solving SVM Using ADMM

(16) Regression

(a) Ridge Regression; Learning a Linear Function
(b) Ridge Regression; Learning an Affine Function
(c) Lasso Regression; Learning a Linear Function
(d) Lasso Regression; Learning an Affine Function
(e) Elastic Net Regression; Learning an Affine Function

(17) Introduction to Deep Learning

(a) Deep Neural Nets
(b) Stochastic Gradient Descent
(c) Back Propagation and the Chain Rule
(d) Convolutional Neural Networks