Problem B1 (40 pts). Refer to Problem B2 of HW6. Write a Matlab program to compute a logarithm $B$ of a rotation matrix $R \in \text{SO}(3)$, namely a skew-symmetric matrix $B$ such that $e^B = R$, using the method described in Problem B2. Write your own program, do not use the built-in Matlab function. In particular, compute a log $B$ of $R$ such that the angle $\theta$ associated with $B$ satisfies the condition $0 \leq \theta \leq \pi$. If $\text{tr}(R) \neq -1$, then $\theta < \pi$ and the matrix $B$ associated with $\theta$ is uniquely determined. Otherwise $\theta = \pi$ and $B$ is determined up to sign.

Problem B2 (210 pts). Refer to Problem B3 of HW6. Similitudes can be used to describe certain deformations (or flows) of a deformable body $B_t$ in 3D. Given some initial shape $B$ in $\mathbb{R}^3$ (for example, a sphere, a cube, etc.), a deformation of $B$ is given by a piecewise differentiable curve

$$D : [0, T] \to \text{SIM}(3),$$

where each $D(t)$ is a similitude (for some $T > 0$). The deformed body $B_t$ at time $t$ is given by

$$B_t = D(t)(B),$$

where $D(t) \in \text{SIM}(3)$ is a similitude.

The surjectivity of the exponential map $\exp : \text{sim}(3) \to \text{SIM}(3)$ implies that there is a map $\log : \text{SIM}(3) \to \text{sim}(3)$, although it is multivalued. The exponential map and the log “function” allows us to work in the simpler (noncurved) Euclidean space $\text{sim}(3)$ (which has dimension 7).

For instance, given two similitudes $A_1, A_2 \in \text{SIM}(3)$ specifying the shape of $B$ at two different times, we can compute $\log(A_1)$ and $\log(A_2)$, which are just elements of the Euclidean space $\text{sim}(3)$, form the linear interpolant $(1-t)\log(A_1) + t\log(A_2)$, and then apply the exponential map to get an interpolating deformation

$$t \mapsto e^{(1-t)\log(A_1) + t\log(A_2)}, \quad t \in [0, 1].$$

Also, given a sequence of “snapshots” of the deformable body $B$, say $A_0, A_1, \ldots, A_m$, where each is $A_i$ is a similitude, we can try to find an interpolating deformation (a curve
in $\text{SIM}(3)$ by finding a simpler curve $t \mapsto C(t)$ in $\text{sim}(3)$ (say, a $B$-spline) interpolating $\log A_1, \log A_1, \ldots, \log A_m$. Then, the curve $t \mapsto e^{C(t)}$ yields a deformation in $\text{SIM}(3)$ interpolating $A_0, A_1, \ldots, A_m$.

(1) (60 pts). Write a program interpolating between two deformations, using the formulae found in Problems B2 and B3 of HW6 (not the built-in Matlab functions!).

(2) (150 pts). Write a program using your cubic spline interpolation program from the first project, to interpolate a sequence of deformations given by similitudes $A_0, A_1, \ldots, A_m$ in $\text{SIM}(3)$. Use the formulae found in Problems B2 and B3 of HW6 (not the built-in Matlab functions!).

TOTAL: 250 pts.