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THE p -FUNCTION IN λ - K -CONVERSION

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In the theory of conversion it is important to have a formally defined function which assigns to any positive integer n the least integer not less than n which has a given property. The definition of such a formula is somewhat involved.¹ I propose to give the corresponding formula in λ - K -conversion,² which will (naturally) be much simpler. I shall in fact find a formula p such that if T be a formula for which $T(n)$ is convertible³ to a formula representing a natural number, whenever n represents a natural number, then $p(T, r)$ is convertible to the formula q representing the least natural number q , not less than r , for which $T(q) \text{ conv } 0$.² The method depends on finding a formula Θ with the property that $\Theta \text{ conv } \lambda u. u(\Theta(u))$, and consequently if $M \rightarrow \Theta(V)$ then $M \text{ conv } V(M)$. A formula with this property is,

$$\Theta \rightarrow \{\lambda v u. u(v(u))\}(\lambda v u. u(v(u))).$$

The formula p will have the required property if $p(T, r) \text{ conv } r$ when $T(r) \text{ conv } 0$, and $p(T, r) \text{ conv } p(T, S(r))$ otherwise. These conditions will be satisfied if $p(T, r) \text{ conv } T(r, \lambda x. p(T, S(r)), r)$, i.e. if $p \text{ conv } \{\lambda p t r. t(r, \lambda x. p(t, S(r)), r)\}(p)$. We therefore put,

$$p \rightarrow \Theta(\lambda p t r. t(r, \lambda x. p(t, S(r)), r)).$$

This enables us to define also a formula,

$$P \rightarrow \lambda t n. n(\lambda v. p(t, S(v)), 0),$$

such that $P(T, n)$ is convertible to the formula representing the n th positive integer q for which $T(q) \text{ conv } 0$.

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¹ Such a function was first defined by S. C. Kleene, *A theory of positive integers in formal logic*, *American journal of mathematics*, vol. 57 (1934), see p. 231.

² For the definition of λ - K -conversion see S. C. Kleene, *λ -definability and recursiveness*, *Duke mathematical journal*, vol. 2 (1936), pp. 340-353, footnote 12. In λ - K -conversion we are able to define the formula $0 \rightarrow \lambda f x. x$. The same paper of Kleene contains the definition of a formula L with a property similar to the essential property of Θ (p. 346).

³ "Convertible" and "conv" refer to λ - K -conversion throughout this note.