Spring, 2014 CIS 511

Introduction to the Theory of Computation
Jean Gallier

Practice Final Exam

April 25, 2014

Note that this is a **closed-book exam**.
Do the exam in 120 mn.

Solutions will be posted on the Web on April 30.

Problem 1 (10 pts). Given an alphabet Σ, sketch an algorithm to decide whether

$$(R^* + S^*) \cong \Sigma^*,$$

for any two regular expressions R and S over Σ.

Problem 2 (20 pts). Let $D = (Q, \Sigma, \delta, q_0, F)$ be any DFA. For any integer, $m \geq 0$, give an algorithm to decide the following questions:

(1) The DFA, D, only accepts strings of length $m \geq 0$.

(2) The DFA, D, only accepts strings of length $\leq m$.

Problem 3 (20 pts). Prove that the following languages are not regular:

$$\begin{aligned} L_1 &= \{a^m b^n c^m \mid m, n \geq 1\}, \\ L_2 &= \{a^n \mid n \text{ is not a prime}\}. \end{aligned}$$

Problem 4 (10 pts). Consider the grammar G with nonterminal set $\{S, A, C\}$ and terminal set $\{a, b, c\}$ given by the following productions:

$$\begin{aligned} S &\Longrightarrow AC \\ A &\Longrightarrow aAb \\ A &\Longrightarrow ab \\ C &\Longrightarrow c. \end{aligned}$$

(i) Describe all rightmost derivations and find out what the set of characteristic strings C_G is.

(ii) Describe all leftmost derivations.

Problem 5 (10 pts). Given any trim DFA $D = (Q, \Sigma, \delta, q_0, F)$ accepting a language $L = L(D)$, if D is a minimal DFA, then prove that its Myhill-Nerode equivalence relation $\simeq_D$ is equal to ρ_L.

Problem 6 (20 pts). Give an algorithm to decide whether $L \subseteq \{aa, bb\}^*$, where $L \subseteq \{a, b\}^*$ is any context-free language.

Problem 7 (20 pts). Prove that the following sets are not recursive ($\varphi_1, \varphi_2, \ldots, \varphi_i, \ldots$ is any acceptable indexing of the partial recursive functions):

$$
\begin{aligned}
A &= \{i \in \mathbb{N} \mid \varphi_i(0) = \varphi_i(1) \quad \text{and} \quad \varphi_i(0), \varphi_i(1) \text{ are both defined}\} \\
B &= \{i \in \mathbb{N} \mid \varphi_i = \varphi_a + \varphi_b\} \\
C &= \{\langle i, j, k\rangle \in \mathbb{N} \mid \varphi_i = \varphi_j + \varphi_k\} \\
D &= \{i \in \mathbb{N} \mid \varphi_i \text{ diverges for exactly one input}\}
\end{aligned}
$$

where a and b are two fixed natural numbers.

Problem 8 (10 pts). Consider the version of the tiling problem where an instance of the problem is of the form $((\mathcal{T}, V, H), \widehat{s}, \sigma_0)$, where $(\mathcal{T}, V, H)$ is a tiling system as before, but $\widehat{s}$ is s *in tally notation* and σ_0 is a single tile placed in position $(1, 1)$. Recall that the tally notation of a natural number n is $1 \cdots 1$ with $n + 1$ occurrences of "1". For example, the tally notation for the decimal number 10 is 11111111111.

Prove that this new tiling problem is also $\mathcal{NP}$-complete.