

CIS 511 Spring 2008: Homework 5, Due April 24

Please write your answers succinctly and rigorously.

1. Determine whether each of the following problems is recursive, RE-but-not-recursive, or non-RE. Justify your answers. 10pts
 - (a) Given a TM M and an integer k , does $L(M)$ contain at least k words?
 - (b) Given a TM M , is $L(M)$ a regular language?

2. Show that the following problems are not recursively enumerable: 10pts
 - (a) The set of pairs $(\langle M_1 \rangle, \langle M_2 \rangle)$ (where M_1, M_2 are TMs) such that $L(M_1) \cap L(M_2) = \emptyset$.
 - (b) The set of triples $(\langle M_1 \rangle, \langle M_2 \rangle, \langle M_3 \rangle)$ (where M_1, M_2, M_3 are TMs) such that $L(M_1) = L(M_2)L(M_3)$; i.e. the language of the first is the concatenation of the other two TMs.

3. Given a graph G with vertices V and edges E , and a positive number k , the MAX-CUT problem is to determine if there exists a subset U of vertices such that there are exactly k edges with one endpoint in U and with one endpoint not in U (that is, the set $\{(u, v) \in E \mid u \in U \text{ and } v \notin U\}$ is of size k). Show that MAX-CUT is polynomial-time reducible to SAT (note: do not rely on the fact that SAT is NP-hard, but give the reduction explicitly). 15pts

4. A subset U of vertices of a graph G is a *dominating set* if every vertex not in U is connected to some vertex in U . Given a graph G and a positive number k , the DOM-SET problem is to determine if G has a dominating set of size k . Show that DOM-SET is NP-complete. You can assume that the node-cover problem (NC) is NP-complete (see Sec 10.4.3 of the textbook). 15pts