

CIS 511 Spring 2008: Homework 3, Due March 4

Please write your answers succinctly and rigorously.

1. Consider the language $L = \{a^n b^m \mid m \neq n \text{ and } m \neq 2n\}$. 10pts
 - (a) Give a pushdown automaton for L .
 - (b) Give a context-free grammar for L .
2. For a language L , let $perm(L)$ be the set of words that are permutations of words in L . For example, if $L = a^*b^*$, then $perm(L) = \{a, b\}^*$, and if $L = \{a^n b^n \mid n \geq 0\}$, then $perm(L)$ contains all words with equal number of a 's and b 's. 15pts
 - (a) Give an example of a regular language L over $\{a, b\}$ such that $perm(L)$ is not regular.
 - (b) Prove that if L is a regular language over $\{a, b\}$, then $perm(L)$ is a context-free language.
3. Consider the grammar G over the alphabet $\{0, 1\}$ with a single non-terminal S and productions given by: 15pts
$$S \rightarrow S1S0 \mid S0 \mid \varepsilon$$
 - (a) What is the language generated by G ?
 - (b) Show that G is ambiguous.
 - (c) Find a grammar G' such that $L(G) = L(G')$ and G' is unambiguous.
4. Using the Pumping Lemma for context-free languages, prove that the following language is not a CFL: 10pts

$$L = \{0^i 1^j \mid j \neq 0 \text{ modulo } i\}$$