Spring, 2004 CIS 511

Introduction to the Theory of Computation
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Practice Final Exam

April 20, 2004

Note that this is a **closed-book exam**. Do the exam in 1 hour and 50 mn.

Solutions will be posted on the Web on Monday, April 26.

Problem 1 (10 pts). Given an alphabet Σ, sketch an algorithm to decide whether

$$R^*S^* = \Sigma^*,$$

for any two regular expressions R and S over Σ.

Problem 2 (20 pts). Let Σ be an alphabet and let R be some regular language over Σ. Recall that for any string $w \in \Sigma^*$, the reversal of w is denoted by w^R. Which of the following languages are regular:

$$\begin{aligned} L_1 &= \{ww^R \mid w \in R\} \\ L_2 &= \{w \in \Sigma^* \mid ww^R \in R\}. \end{aligned}$$

Justify your answer carefully.

Problem 3 (15 pts). (i) Give a context-free grammar for the language:

$$L_3 = \{a^m b^m c a^{2n} b^{2n} \mid m, n \geq 1\},$$

where $\Sigma = \{a, b, c\}$.

(ii) Give a *deterministic* PDA accepting L_3.

Problem 4 (20 pts). Prove that the following languages are not context-free:

$$\begin{aligned} L_4 &= \{a^m b^n c a^{2m} b^{2n} \mid m, n \geq 1\}, \\ L_5 &= \{a^n \mid n \text{ is not a prime}\}. \end{aligned}$$

Problem 5 (20 pts). A *linear context-free grammar* is a context-free grammar whose productions are of the form either

$$\begin{aligned} A &\longrightarrow uBv, \quad \text{or} \\ A &\longrightarrow u, \end{aligned}$$

where A, B are nonterminals and $u, v \in \Sigma^*$. A language is *linear context-free* iff it is generated by some linear context-free grammar.

Prove that every regular language is linear context-free. Prove that if L is a linear context-free language, then for every $a \in \Sigma$, the language $L/a = \{w \in \Sigma^* \mid wa \in L\}$ is also linear context-free.
Hint. Construct a grammar using some new nonterminals, $[A/a]$, and new productions

$$[A/a] \longrightarrow \alpha, \quad \text{if} \quad A \longrightarrow \alpha a \in P \quad \text{or} \quad A \longrightarrow \alpha a B \in P \quad \text{with} \quad B \stackrel{+}{\Longrightarrow} \epsilon$$

and

$$[A/a] \longrightarrow u[B/a], \quad \text{if} \quad A \longrightarrow uB \in P,$$

Problem 6 (25 pts). Any Turing machine, M, can easily be altered by adding two states without otherwise modifying the computations performed by M. This modification insures that every computation of M halting in a proper ID has at least three ID's, i.e., is of the form

$$w_1\#w_2^R\#w_3\#\cdots$$

(a) If L_M is the language of computations halting in a proper ID, prove that L_M is context-free iff M halts for only finitely many inputs.

(b) Prove that it is undecidable for an arbitrary context-free grammar, G, whether $\overline{L(G)}$ is context-free.