Spring, 2002 CIS 511

Introduction to the Theory of Computation

Final Exam

April 30, 2002

Note that this is a **closed-book exam**
Read all the questions **before** starting solving any of them!
If you are taking the WPE, please write your ID number on **all** your solution sheets.

Problem 1 (10 pts). Given an alphabet, Σ, sketch an algorithm to decide whether any regular expression, R, is equivalent to some regular expression, S, that is star-free (i.e., S is built up from the atoms using only $+$ and $\cdot$).

Problem 2 (20 pts). Let Σ be an alphabet and let R be some regular language over $\Sigma \cup \{c\}$, where $c \notin \Sigma$. Which of the following languages are regular?

$$\begin{aligned} L_1 &= \{wcw \mid w \in R \cap \Sigma^*\} \\ L_2 &= \{w \in \Sigma^* \mid wcw \in R\}. \end{aligned}$$

Justify your answers carefully.

Problem 3 (15 pts). (i) Give a context-free grammar for the language

$$L_3 = \{uu^R cvv^R \mid u, v \in \{a, b\}^*\}.$$

(ii) Give a PDA accepting L_3. Can you give a DPDA? (your answer does not have to be very formal, just give the main idea).

Problem 4 (25 pts). Prove that the following languages are not context-free:

$$\begin{aligned} L_4 &= \{a^{(n+1)^2} \mid n \geq 1\}, \\ L_5 &= \{baba^2ba^3b \cdots ba^nb \mid n \geq 1\}. \end{aligned}$$

Problem 5 (25 pts). For any language, $L \subseteq \Sigma^*$, and any $u \in \Sigma^*$, let

$$u/L = \{v \in \Sigma^* \mid uv \in L\}.$$

(i) Prove that if L is context-free, then u/L is also context-free.

Hint. First, consider $u = a \in \Sigma$, and use the Greibach normal form.

(ii) For any language, $L \subseteq \Sigma^*$, and any $v \in \Sigma^*$, let

$$L/v = \{u \in \Sigma^* \mid uv \in L\}.$$

Use the fact (which you **do not** need to prove) that the context-free languages are closed under reversal to prove that if L is context-free, then L/v is also context-free.

(iii) Prove that it is undecidable for any sufficiently large alphabet, Σ, and any context-free language, L, whether $\overline{L} = \Sigma^* - L$ is context-free.

Problem 6 (15 pts). Given any context-free language, L, and any regular language, R, are the following problems decidable:

(1) $L \subseteq R$?

(2) $R \subseteq L$?

Justify your answers.