

Problem 1 (10 points):

State whether each of the following is true or false:

1. $\phi \in \phi$
2. $\phi \subseteq \phi$
3. $\phi \in \{\phi\}$
4. $\{a, b\} \in P\{a, b, \{a, b\}\}$
5. $\{a, b\} \subseteq P\{a, b, \{a, b\}\}$

Problem 2 (10 points):

Let $f: A \rightarrow B$, and let R be a relation defined as follows:

$\langle a, b \rangle \in R$ if and only if $f(a) = f(b)$.

Show that R is an equivalence relation.

Problem 3 (25 points):

Show that in any group of people there are at least two persons that have the same number of acquaintances within the group. (Hint: Use the notion of one-to one correspondence.)

Problem 4 (20 points):

Let R be a binary relation. A chain in R is a sequence (a_1, a_2, \dots, a_n) , $n \geq 1$ such that $\langle a_i, a_{i+1} \rangle \in R$, $i=1, 2, \dots, n-1$. A chain (a_1, a_2, \dots, a_n) is a cycle if the a_i are all distinct and $\langle a_n, a_1 \rangle \in R$. A cycle (a_1, a_2, \dots, a_n) is trivial if $n=1$, otherwise it is non-trivial.

Show that a relation is a partial order if and only if it is reflexive and transitive and has no nontrivial cycles.

Problem 5 (15 points):

Let A be the set of all English words. Let R be a relation such that $\langle a, b \rangle \in R$ if and only if a is longer than b .

- Is R a partial order?
- a linear order?
- an equivalence relation?

Problem 6 (20 points):

Construct a finite state automaton M (i.e., the state diagram) with $\Sigma = \{a, b\}$ and $L(M) =$ the set of all strings that do NOT contain aab .

If M is nondeterministic then construct M' such that M' is deterministic and $L(M) = L(M')$.