1.4 Arrays
A Foundation for Programming

any program you might want to write

objects

functions and modules

graphics, sound, and image I/O

arrays

conditionals and loops

Math  text I/O

primitive data types  assignment statements

store and manipulate huge quantities of data
Arrays

This lecture Store and manipulate huge quantities of data

Array Indexed sequence of values of the same type

Examples

- 52 playing cards in a deck
- 10 thousand undergrads at Penn
- 1 million characters in a book
- 10 million audio samples in an MP3 file
- 4 billion nucleotides in a DNA strand
- 73 billion Google queries per year
- 50 trillion cells in the human body
- $6.02 \times 10^{23}$ particles in a mole

<table>
<thead>
<tr>
<th>index</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>bjbrown</td>
</tr>
<tr>
<td>1</td>
<td>hawka</td>
</tr>
<tr>
<td>2</td>
<td>apathare</td>
</tr>
<tr>
<td>3</td>
<td>anann</td>
</tr>
<tr>
<td>4</td>
<td>catheriz</td>
</tr>
<tr>
<td>5</td>
<td>ericslee</td>
</tr>
<tr>
<td>6</td>
<td>fannliu</td>
</tr>
<tr>
<td>7</td>
<td>lilleyia</td>
</tr>
</tbody>
</table>
Many Variables of the Same Type

*Goal* 10 variables of the same type

```c
// tedious and error-prone
double a0, a1, a2, a3, a4, a5, a6, a7, a8, a9;
a0 = 0.0;
a1 = 0.0;
a2 = 0.0;
a3 = 0.0;
a4 = 0.0;
a5 = 0.0;
a6 = 0.0;
a7 = 0.0;
a8 = 0.0;
a9 = 0.0;
...
a4 = 3.0;
...
a8 = 8.0;
...
double x = a4 + a8;
```
Many Variables of the Same Type

**Goal** 10 variables of the same type

```java
// easy alternative
double[] a = new double[10];
...
a[4] = 3.0;
...
a[8] = 8.0;
...
double x = a[4] + a[8];
```
Many Variables of the Same Type

**Goal** 1 million variables of the same type

```java
// scales to handle large arrays
double[] a = new double[1000000];
...
a[123456] = 3.0;
...
a[987654] = 8.0;
...
double x = a[123456] + a[987654];
```

delares, creates, and initializes
[stay tuned for details]
Arrays in Java

Java has special language support for arrays

- To make an array: declare, create, and initialize it
- To access entry $i$ of array named $a$, use $a[i]$
- Array indices start at 0

```java
int N = 10; // size of array
double[] a; // declare the array
a = new double[N]; // create the array
for (int i = 0; i < N; i++) // initialize the array
  a[i] = 0.0; // all to 0.0
```
Arrays in Java

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    a[i] = 0.0; // all to 0.0
```

Compact alternative

- Declare, create, and initialize in one statement
- Default initialization: all numbers automatically set to zero

```java
int N = 10; // size of array
double[] a = new double[N]; // declare, create, init
```
Dot product  Given two vectors \( \mathbf{x} \) and \( \mathbf{y} \) of length \( N \), their dot product is the sum of the products of their corresponding components.

```java
double[] x = { 0.3, 0.6, 0.1 };
double[] y = { 0.5, 0.1, 0.4 };
int N = x.length;
double sum = 0.0;
for (int i = 0; i < N; i++) {
    sum = sum + x[i]*y[i];
}
```

<table>
<thead>
<tr>
<th>i</th>
<th>( x[i] )</th>
<th>( y[i] )</th>
<th>( x[i]*y[i] )</th>
<th>sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.30</td>
<td>0.50</td>
<td>0.15</td>
<td>0.15</td>
</tr>
<tr>
<td>1</td>
<td>0.60</td>
<td>0.10</td>
<td>0.06</td>
<td>0.21</td>
</tr>
<tr>
<td>2</td>
<td>0.10</td>
<td>0.40</td>
<td>0.04</td>
<td>0.25</td>
</tr>
</tbody>
</table>

The dot product of \( \mathbf{x} \) and \( \mathbf{y} \) is 0.25.
## Array-Processing Examples

<table>
<thead>
<tr>
<th>Task</th>
<th>Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>create an array with random values</td>
<td><code>double[] a = new double[N];</code>&lt;br&gt;<code>for (int i = 0; i &lt; N; i++)</code>&lt;br&gt;<code>a[i] = Math.random();</code></td>
</tr>
<tr>
<td>print the array values, one per line</td>
<td><code>for (int i = 0; i &lt; N; i++)</code>&lt;br&gt;<code>System.out.println(a[i]);</code></td>
</tr>
<tr>
<td>find the maximum of the array values</td>
<td><code>double max = Double.NEGATIVE_INFINITY;</code>&lt;br&gt;<code>for (int i = 0; i &lt; N; i++)</code>&lt;br&gt;<code>if (a[i] &gt; max) max = a[i];</code></td>
</tr>
<tr>
<td>compute the average of the array values</td>
<td><code>double sum = 0.0;</code>&lt;br&gt;<code>for (int i = 0; i &lt; N; i++)</code>&lt;br&gt;<code>sum += a[i];</code>&lt;br&gt;<code>double average = sum / N;</code></td>
</tr>
<tr>
<td>copy to another array</td>
<td><code>double[] b = new double[N];</code>&lt;br&gt;<code>for (int i = 0; i &lt; N; i++)</code>&lt;br&gt;<code>b[i] = a[i];</code></td>
</tr>
<tr>
<td>reverse the elements within an array</td>
<td><code>for (int i = 0; i &lt; N/2; i++)</code>&lt;br&gt;<code>{</code>&lt;br&gt;<code>  double temp = b[i];</code>&lt;br&gt;<code>  b[i] = b[N-1-i];</code>&lt;br&gt;<code>  b[N-i-1] = temp;</code>&lt;br&gt;<code>}</code></td>
</tr>
</tbody>
</table>
Shuffling a Deck
Ex. Print a random card

```java
String[] rank = {
    "2", "3", "4", "5", "6", "7", "8", "9",
    "10", "Jack", "Queen", "King", "Ace"
};

String[] suit = {
    "Clubs", "Diamonds", "Hearts", "Spades"
};

int i = (int) (Math.random() * 13); // between 0 and 12
int j = (int) (Math.random() * 4); // between 0 and 3

System.out.println(rank[i] + " of " + suit[j]);
```
Setting Array Values at Run Time

Ex. Create a deck of playing cards and print them out

```java
String[] deck = new String[52];
for (int i = 0; i < 13; i++)
    for (int j = 0; j < 4; j++)
        deck[4*i + j] = rank[i] + " of " + suit[j];

for (int i = 0; i < 52; i++)
    System.out.println(deck[i]);
```

Q. In what order does it output them?

A. 
- two of clubs
- two of diamonds
- two of hearts
- two of spades
- three of clubs
...

B. 
- two of clubs
- three of clubs
- four of clubs
- five of clubs
- six of clubs
- ...

Typical array-processing code changes values at runtime.
Shuffling

Goal Given an array, rearrange its elements in random order

Shuffling algorithm
  - In iteration \(i\), pick random card from \(\text{deck}[i]\) through \(\text{deck}[N-1]\), with each card equally likely
  - Exchange it with \(\text{deck}[i]\)

```java
int N = deck.length;
for (int i = 0; i < N; i++) {
    int r = i + (int) (Math.random() * (N-i));
    String t = deck[r];
    deck[r] = deck[i];
    deck[i] = t;
}
```
Shuffling a Deck of Cards: Putting Everything Together

```java
public class Deck {
    public static void main(String[] args) {
        String[] suit = { "Clubs", "Diamonds", "Hearts", "Spades" };
        String[] rank = { "2", "3", "4", "5", "6", "7", "8", "9", "10", "Jack", "Queen", "King", "Ace" };
        int SUITS = suit.length;
        int RANKS = rank.length;
        int N = SUITS * RANKS;

        String[] deck = new String[N];
        for (int i = 0; i < RANKS; i++) {
            for (int j = 0; j < SUITS; j++)
                deck[SUITS*i + j] = rank[i] + " of " + suit[j];
        }

        for (int i = 0; i < N; i++) {
            int r = i + (int) (Math.random() * (N-i));
            String t = deck[r];
            deck[r] = deck[i];
            deck[i] = t;
        }

        for (int i = 0; i < N; i++)
            System.out.println(deck[i]);
    }
}
```
Shuffling a Deck of Cards

% java Deck
5 of Clubs
Jack of Hearts
9 of Spades
10 of Spades
9 of Clubs
7 of Spades
6 of Diamonds
7 of Hearts
7 of Clubs
4 of Spades
Queen of Diamonds
10 of Hearts
5 of Diamonds
Jack of Clubs
Ace of Hearts
...
5 of Spades

% java Deck
10 of Diamonds
King of Spades
2 of Spades
3 of Clubs
4 of Spades
Queen of Clubs
2 of Hearts
7 of Diamonds
6 of Spades
Queen of Spades
3 of Spades
Jack of Diamonds
6 of Diamonds
8 of Spades
9 of Diamonds
...
10 of Spades
Texas hold 'em poker. **Software must shuffle electronic deck of cards**

How we learned to cheat at online poker: a study in software security

http://itmanagement.earthweb.com/entdev/article.php/616221
Coupon Collector
Coupon Collector Problem

**Coupon collector problem** Given \( N \) different card types, how many do you have to collect before you have (at least) one of each type?

Assuming each possibility is equally likely for each card that you collect.

**Simulation algorithm** Repeatedly choose an integer \( i \) between 0 and \( N-1 \)
Stop when we have at least one card of every type

**Q.** How to check if we've seen a card of type \( i \)?
**A.** Maintain a boolean array so that \( \text{found}[i] \) is true if we've already collected a card of type \( i \)
public class CouponCollector {
    public static void main(String[] args) {
        int N = Integer.parseInt(args[0]);
        int cardcnt = 0; // number of cards collected
        int valcnt = 0; // number of distinct cards

        // do simulation
        boolean[] found = new boolean[N];
        while (valcnt < N) {
            int val = (int) (Math.random() * N);
            cardcnt++;
            if (!found[val]) {
                valcnt++;
                found[val] = true;
            }
        }

        // all N distinct cards found
        System.out.println(cardcnt);
    }
}
Coupon Collector: Debugging

**Debugging**  Add code to print contents of all variables

<table>
<thead>
<tr>
<th>val</th>
<th>found</th>
<th>valcnt</th>
<th>cardcnt</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0 F</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1 F</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>2 T</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>3 F</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>0</td>
<td>3 T</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>4 T</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>4 T</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>5 T</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>0</td>
<td>5 T</td>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>1</td>
<td>5 T</td>
<td>5</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>6 T</td>
<td>6</td>
<td>10</td>
</tr>
</tbody>
</table>

**Challenge**  Debugging with arrays requires tracing many variables
Coupon Collector: Mathematical Context

**Coupon collector problem**  Given N different possible cards, how many do you have to collect before you have (at least) one of each type?

**Fact**  About N \( (1 + 1/2 + 1/3 + \ldots + 1/N) \sim N \ln N \)

**Ex.**  N = 30 baseball teams. Expect to wait \( \approx 120 \) years before all teams win a World Series  

under idealized assumptions
Q. Given a sequence from nature, does it have same characteristics as a random sequence?

A. No easy answer - many tests have been developed

**Coupon collector test** Compare number of elements that need to be examined before all values are found against the corresponding answer for a random sequence
Multidimensional Arrays
Two-Dimensional Arrays

Two-dimensional arrays
- Table of data for each experiment and outcome
- Table of grades for each student and assignments
- Table of grayscale values for each pixel in a 2D image

Mathematical abstraction  Matrix
Java abstraction  2D array

Reference: Botstein & Brown group
Two-Dimensional Arrays in Java

**Array access** Use \( a[i][j] \) to access entry in row \( i \) and column \( j \)

**Zero-based indexing** Row and column indices start at 0

```java
int M = 10;
int N = 3;
double[][] a = new double[M][N];
for (int i = 0; i < M; i++) {
    for (int j = 0; j < N; j++) {
        a[i][j] = 0.0;
    }
}
```

A 10-by-3 array
Setting 2D Array Values at Compile Time

Initialize 2D array by listing values

```c
double[][] p = {
    { .02, .92, .02, .02, .02 },
    { .02, .02, .32, .32, .32 },
    { .02, .02, .02, .92, .02 },
    { .92, .02, .02, .02, .02 },
    { .47, .02, .47, .02, .02 },
};
```
Matrix Addition

Matrix addition Given two N-by-N matrices $a$ and $b$, define $c$ to be the N-by-N matrix where $c[i][j]$ is the sum $a[i][j] + b[i][j]$

double[][] c = new double[N][N];
for (int i = 0; i < N; i++)
    for (int j = 0; j < N; j++)
        c[i][j] = a[i][j] + b[i][j];
Matrix Multiplication

Matrix multiplication: Given two N-by-N matrices $a$ and $b$, define $c$ to be the N-by-N matrix where $c[i][j]$ is the dot product of the $i^{th}$ row of $a[][]$ and the $j^{th}$ column of $b[][]$.

```java
double[][] c = new double[N][N];
for (int i = 0; i < N; i++)
    for (int j = 0; j < N; j++)
        for (int k = 0; k < N; k++)
            c[i][j] += a[i][k] * b[k][j];
```

All values initialized to 0

dot product of row $i$ of $a[][]$ and column $j$ of $b[][]$

Example:

- $a[][] = \begin{bmatrix} .70 & .20 & .10 \\ .30 & .60 & .10 \\ .50 & .10 & .40 \end{bmatrix}$
- $b[][] = \begin{bmatrix} .80 & .30 & .50 \\ .10 & .40 & .10 \\ .10 & .30 & .40 \end{bmatrix}$
- $c[][] = \begin{bmatrix} .59 & .32 & .41 \\ .31 & .36 & .25 \\ .45 & .31 & .42 \end{bmatrix}$
Array Challenge

Q. How many scalar multiplications multiply two N-by-N matrices?

A. N  B. N^2  C. N^3  D. N^4

double[][] c = new double[N][N];
for (int i = 0; i < N; i++)
  for (int j = 0; j < N; j++)
    for (int k = 0; k < N; k++)
      c[i][j] += a[i][k] * b[k][j];
Summary

Arrays

- Organized way to store huge quantities of data
- Almost as easy to use as primitive types
- Can directly access an element given its index

Ahead  Reading in large quantities of data from a file into an array

http://imgs.xkcd.com/comics/donald_knuth.png