Advanced Programming Handout 9

Qualified Types

(SOE Chapter 12)

Motivation

- What should the principal type of (+) be?
  - `Int -> Int -> Int` -- too specific
  - `a -> a -> a` -- too general
- It seems like we need something "in between", that restricts "a" to be from the set of all number types, say Num = {Int, Integer, Float, Double, etc.}.
- The type a -> a -> a is really shorthand for (∀ a) a -> a -> a
- *Qualified types* generalize this by qualifying the type variable, as in (∀ a ∈ Num) a -> a -> a, which in Haskell we write as Num a => a -> a -> a

Type Classes

- "Num" in the previous example is called a *type class*, and should not be confused with a type constructor or a value constructor.
- "Num T" should be read "T is a member of (or an instance of) the type class Num".
- Haskell's type classes are one of its most innovative features.
- This capability is also called "overloading", because one function name is used for potentially very different purposes.
- There are many pre-defined type classes, but you can also define your own.

Example: Equality

- Like addition, equality is not defined on all types (how would we test the equality of two functions, for example?).
- So the equality operator (==) in Haskell has type Eq a => a -> a -> Bool. For example:

42 == 42	➔ True
'a' == 'a'	➔ True
'a' == 42	➔ << type error! >> (types don't match)
(+1) == (\x->x+1)	➔ << type error! >> ((->) is not an instance of Eq)

- Note: the type errors occur *at compile time!*

Equality, cont'd

- Eq is defined by this *type class declaration:*

```
class Eq a  where
    (==), (/=)      :: a -> a -> Bool
    x /= y          =  not (x == y)
    x == y          =  not (x /= y)
```

- The last two lines are *default methods* for the operators defined to be in this class.
- A type is made an instance of a class by an *instance declaration.* For example:

```
instance Eq Int where
    x == y = intEq x y   -- primitive equality for Ints
instance Eq Float where
    x == y = floatEq x y -- primitive equality for Floats
```

Equality, cont'd

- *User-defined* data types can also be made instances of Eq. For example:

```
data Tree a = Leaf a | Branch (Tree a) (Tree a)
instance         Eq (Tree a) where
    Leaf a1      == Leaf a2      = a1 == a2
    Branch l1 r1 == Branch l2 r2 = l1==l2 && r1==r2
    _            == _            = False
```

- But something is strange here: is "`a1 == a2`" on the right-hand side correct? How do we know that equality is defined on the type "`a`"???

Equality, cont'd

- *User-defined* data types can also be made instances of Eq. For example:

```
data Tree a = Leaf a | Branch (Tree a) (Tree a)

instance Eq a => Eq (Tree a) where
    Leaf a1       == Leaf a2       = a1 == a2
    Branch l1 r1 == Branch l2 r2 = l1==l2 && r1==r2
    _             == _             = False
```

- But something is strange here: is "`a1 == a2`" on the right-hand side correct? How do we know that equality is defined on the type "`a`"???
- Answer: Add a constraint that requires `a` to be an equality type.

Constraints / Contexts are Propagated

- Consider this function:

```
x `elem` []      = False
x `elem` (y:ys)  = x==y || x `elem` ys
```

- Note the use of (==) on the right-hand side of the second equation. So the principal type for elem is:

```
elem :: Eq a => a -> [a] -> Bool
```

- This is inferred automatically by Haskell, but, as always, it is recommended that you provide your own type signature for all functions.

Classes for Regions

- Useful slogan:
  "polymorphism captures similar structure over different values, while type classes capture similar operations over different structures."
- For a simple example, recall from Chapter 8:

```
containsS :: Shape -> Point -> Bool
containsR :: Region -> Point -> Bool
```

- These are similar ops over different structures. So:

```
class PC t where
    contains :: t -> Point -> Bool
instance PC Shape where
    contains = containsS
instance PC Region where
    contains = containsR
```

Numeric Classes

- Haskell's numeric types are embedded in a very rich, hierarchical set of type classes.
- For example, the Num class is defined by:

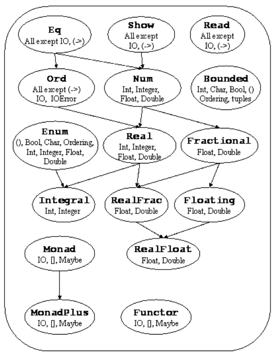
```
class (Eq a, Show a) => Num a where
    (+), (-), (*) :: a -> a -> a
    negate :: a -> a
    abs, signum :: a -> a
    fromInteger :: Integer -> a
```

- ...where Show is a type class whose main operator is

```
show :: Show a => a -> String
```

- See the Numeric Class Hierarchy in the Haskell Report on the next slide.

Haskell's Numeric Class Hierarchy



Coercions

- Note this method in the class Num:

```
fromInteger :: Num a => Integer -> a
```

- Also, in the class Integral:

```
toInteger :: Integral a => a -> Integer
```

- This explains the definition of intToFloat:

```
intToFloat :: Int -> Float
intToFloat n = fromInteger (toInteger n)
```

- These generic coercion functions avoid a quadratic blowup in the number of coercion functions.
- Also, every integer literal, say "42", is really shorthand for "fromInteger 42", thus allowing that number to be typed as *any* member of Num.

Derived Instances

- Instances of the following type classes may be automatically *derived*:
  Eq, Ord, Enum, Bounded, Ix, Read, and Show
- This is done by adding a *deriving* clause to the data declaration. For example:
  ```
  data Tree a = Leaf a | Branch (Tree a) (Tree a)
            deriving (Show, Eq)
  ```
- This will automatically create an instance for Show (Tree a) as well as one for Eq (Tree a) that is precisely equivalent to the one we defined earlier.

Derived vs. User-Defined

- Suppose we define an implementation of finite sets in terms of lists, like this:

```
data Set a = Set [a]

insert (Set s) x = Set (x:s)

member (Set s) x = elem x s

union (Set s) (Set t) = Set (s++t)
```

Derived vs. User-Defined

- We can automatically derive an equality function just by adding “deriving Eq” to the declaration.

```
data Set a = Set [a]
   deriving Eq

insert (Set s) x = Set (x:s)

member (Set s) x = elem x s

union (Set s) (Set t) = Set (s++t)
```

But is this really what we want??

Derived vs. User-Defined

- No!
- E.g.,
  (Set [1,2,3]) == (Set [1,1,2,2,3,3]) ➔ False

A Better Way

```
data Set a = Set [a]

instance Eq a => Eq (Set a) where
  s == t  =  subset s t && subset t s

subset (Set ss) t = all (member t) ss
```

Reasoning About Type Classes

- Most type classes implicitly carry a set of *laws*.
- For example, the Eq class is expected to obey:
  (a /= b) = not (a == b)
  (a == b) && (b == c) ⊇ (a == c)
- Similarly, for the Ord class:
  a <= a
  (a <= b) && (b <= c) ⊇ (a <= c)
  (a <= b) && (b <= a) ⊇ (a == b)
  (a /= b) ⊇ (a < b) || (b < a)
- These laws capture the properties of an *equivalence class* and a *total order*, respectively.
- Unfortunately, there is nothing in Haskell that *enforces* the laws – its up to the programmer!