Lazy functional programming for real

Tackling the Awkward Squad

Adapted by BCP from original slides by

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Beauty and the Beast

- Functional programming is beautiful, and many books tell us why
- But to write real applications, we must deal with un-beautiful issues "around the edges":
  - Input/output
  - Concurrency
  - Error recovery
  - Foreign-language interfaces

The Awkward Squad

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The direct approach

Do everything in "the usual way" (as in ML, Scheme, etc.)

- **I/O** via "functions" with side effects
  `putchar 'x' + putchar 'y'`
- **Concurrency** via operating system threads; OS calls mapped to "functions"
- **Error recovery** via exceptions
- **Foreign language procedures** mapped to "functions"

But…

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The lazy hair shirt

In a lazy functional language like Haskell, order of evaluation is *deliberately* undefined.

- `putchar 'x' + putchar 'y'`
  Output depends on evaluation order of (+)
- `[putchar 'x', putchar 'y']`
  Output (if any) depends on how the consumer evaluates the list

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Tackling the awkward squad

- So lazy languages force us to take a different, more principled, approach to the Awkward Squad.
- These lectures and the accompanying notes describe that approach in detail for Haskell.

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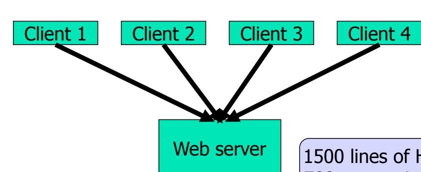
A web server

- We'll use a web server as the motivating example
- Lots of I/O, lots of concurrency, need for error recovery, need to call external libraries



1500 lines of Haskell
700 connections/sec

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Monadic input and output

(review)

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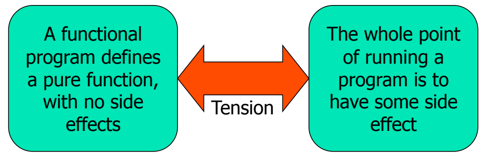
The problem



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Functional I/O

```
main :: [Response] -> [Request]

data Request  = ReadFile Filename
              | WriteFile FileName String
              | …

data Response = RequestFailed
              | ReadOK String
              | WriteOk
              | …
```

- "Wrapper program" interprets requests, and adds responses to input

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Functional I/O is awkward

- Hard to extend (new I/O operations ⇒ new constructors)
- No direct connection between **Request** and corresponding **Response**
- Easy to get "out of step" (⇒ deadlock)

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Monadic I/O: the key idea

A value of type (`IO t`) is an "action" that, when performed, may do some input/output before delivering a result of type t.

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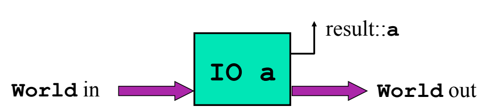
A helpful picture

A value of type (`IO t`) is an "action" that, when performed, may do some input/output before delivering a result of type t.

```
type IO a = World -> (a, World)
```



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Actions are first class

A value of type (`IO t`) is an "action" that, when performed, may do some input/output before delivering a result of type t.

```
type IO a = World -> (a, World)
```

- "Actions" are sometimes called "computations"
- An action is a first class value
- Evaluating an action expression has no effect; performing the resulting action has an effect

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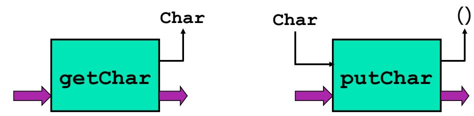
Simple I/O




```
getChar :: IO Char
putChar :: Char -> IO ()

main :: IO ()
main = putChar 'x'
```

Main program is an action of type IO ()

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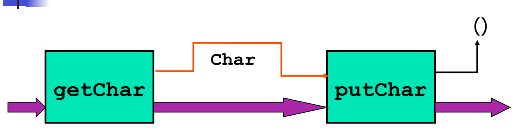
Connecting actions up



Goal: read a character and then write it back out

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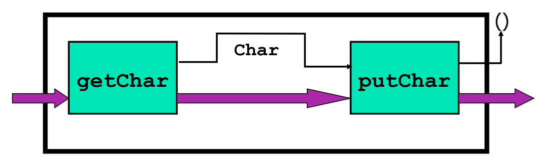
The (>>=) combinator

```
(>>=) :: IO a -> (a -> IO b) -> IO b
```



- We have connected two actions to make a new, bigger action.

```
echo :: IO ()
echo = getChar >>= putChar
```

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Printing a character twice

```
echoDup :: IO ()
echoDup = getChar   >>= (\c ->
          putChar c >>= (\() ->
          putChar c))
```

This is just noise...

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The (>>) combinator

```
echoDup :: IO ()
echoDup = getChar   >>= \c ->
          putChar c >>
          putChar c
```

```
(>>) :: IO a -> IO b -> IO b
m >> n  =  m >>= (\_ -> n)
```

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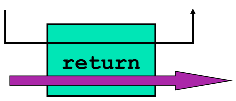
The return combinator

```
getTwoChars :: IO (Char,Char)
getTwoChars = getChar     >>= \c1 ->
              getChar     >>= \c2 ->
              return (c1,c2)
```

return :: a -> IO a



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Notational convenience

```
getTwoChars :: IO (Char,Char)
getTwoChars = getChar    >>= \c1 ->
              getChar    >>= \c2 ->
              return (c1,c2)
```

- By design, the layout looks imperative

```
c1 = getchar();
c2 = getchar();
return (c1,c2);
```

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Notational convenience

```
getTwoChars :: IO (Char,Char)
getTwoChars = do { c1 <-
  getChar ;          c2 <-
  getChar ;          return (c1,c2) }
```

do notation adds only "syntactic sugar"

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Desugaring do notation

do { x<-e; s }	=	e >>= \x -> do { s }
do { e; s }	=	e >> do { s }
do { e }	=	e

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Loops

Values of type (IO t) are first class

So we can define our own "control structures"

```
for :: [a] -> (a -> IO b) -> IO ()
for []     fa = return ()
for (x:xs) fa = fa x >> for xs fa
```

e.g. `for [1..10] (\x -> putStr (show x))`

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First class actions

Slogan: first-class actions let us write application-specific control structures

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What does it all mean?

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What does "mean" mean?

In linguistics, the structure of natural languages is described and studied at many levels…

Phonetics	What basic sounds (phonemes) are possible in a given language
Morphology	How phonemes fit together to make words
Syntax	How words are arranged into grammatical sentences
Semantics	What these sentences mean

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What does "mean" mean?

Programming languages can be described in pretty much the same way…

Phonetics	What basic sounds (phonemes) are possible in a given language	Character set (ASCII)
Morphology	How phonemes fit together to make words	Lexing
Syntax	How words are arranged into grammatical sentences	Parsing
Semantics	What these sentences mean	Semantics

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Semantics of programs

The meaning of programs can be described rigorously (i.e., mathematically) in different ways…

- **Denotational semantics**: The meaning of a program is a mathematical function from inputs to outputs.
- **Operational semantics**: The meaning of a program is the sequence of states that some "abstract machine" goes through when executing it.

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Denotational Semantics

The meaning of an expression of type Int->Int is a function on the set of integers.

foo x = x*2+1 means foo = { ...,
(-2,-3),
(-1,-1),
(0,1),
(1,3),
(2,5),
... }

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Denotational Semantics

This gives us a very natural way to talk about *program equivalence*

foo x = x*2+1

means the same as

foo' x = 1+((1+1)*x)

because both mean { ..., (-2,-3), (-1,-1), (0,1),
(1,3), (2,5), ... }

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Denotational Semantics

The meaning of an expression of type Int->Int is a partial function on the set of integers.

```
fact x =
  if x=0
   then 1
   else x * fact (x-1)
```

means foo = { ...,
(-2,⊥),
(-1,⊥),
(0,1),
(1,1),
(2,2),
(3,6),
... }

pronounced "bottom"

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Denotational Semantics

So the meaning of (fact -2) is ⊥.

I.e., all non-terminating programs mean the same thing.

Intuitively, this makes good sense...
(All we can "observe" about a non-terminating program is that it doesn't terminate!)

...as long as we are only talking about purely functional expressions.

But...

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Denotational semantics of IO?

```
type IO a = World -> (a, World)
```

- A program that loops forever has meaning ⊥.
  A program that prints 'x' forever also has meaning ⊥!
- What is the meaning of two Haskell programs running in parallel?
- Denotational semantics does not scale well to concurrency, non-determinism, etc.

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Operational semantics

Instead of saying what the meaning of a program **is**, say how the program **behaves**

Equivalance of programs becomes **similarity of behaviour** instead of **identity of meaning**

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Operational semantics

General form:

$$P \xrightarrow{\alpha} Q$$

Program state P can move to program state Q, exchanging event α with the environment

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Program states

A program state represents the current internal state of the program.

Initially, it is just a term, {M}

e.g. `{putChar 'x' >> putChar 'y'}`

Notation:
Curly braces = "here is a program state"

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Events

Events describe how the program interacts with the external world: i.e. what I/O it performs

- $P \xrightarrow{!c} Q$ P can move to Q, writing c to stdout
- $P \xrightarrow{?c} Q$ P can move to Q, reading c from stdin

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Our first two rules

$$\{\texttt{putChar}\ ch\} \xrightarrow{!ch} \{\texttt{return}\ ()\}$$
$$\{\texttt{getChar}\} \xrightarrow{?ch} \{\texttt{return}\ ch\}$$

Now, what about this?

`{getChar >>= \c-> putChar c}` $\rightarrow$ **???**

Want to say "look at the action in the leftmost position..."

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Evaluation contexts

E ::= [.] | E >>= M

An **evaluation context** E is a term with a "hole" in it. For example:

E1 = [.] >>= M
E2 = ([.] >>= M1) >>= M2

E[M] is the evaluation context E with the hole filled in by M. So

E1[getChar] = getChar >>= M
E2[getChar] = (getChar >>= M1) >>= M2

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Revised rules for put/get

E ::= [.] | E >>= M

$$\{\mathrm{E}[\texttt{putChar}\ ch]\} \xrightarrow{!ch} \{\mathrm{E}[\texttt{return}\ ()]\}$$
$$\{\mathrm{E}[\texttt{getChar}]\} \xrightarrow{?ch} \{\mathrm{E}[\texttt{return}\ ch]\}$$

`{getChar >>= \c-> putChar c}`
$\xrightarrow{?ch}$ `{return` ***ch*** `>>= \c-> putChar c}`

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The return/bind rule

$$\{\mathrm{E}[\texttt{return}\ N \texttt{>>=} M]\} \rightarrow \{\mathrm{E}[M\ N]\}$$

"Silent transition" (no IO)

`{getChar >>= \c-> putChar c}`
$\xrightarrow{?ch}$ `{return` ***ch*** `>>= \c-> putChar c}`
$\rightarrow$ `{(\c-> putChar c)` ***ch***`}`

Now we need to do some "ordinary evaluation"

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The evaluation rule

V is the value of M

M wasn't already evaluated

"Inference rule" notation:
If the things above the line are true, then we can deduce the thing below the line

$$\frac{\mathcal{E}[\![M]\!] = V \quad M \not\equiv V}{\{\mathrm{E}[M]\} \rightarrow \{\mathrm{E}[V]\}}$$

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The evaluation rule

$$\frac{\mathcal{E}[\![M]\!] = V \qquad M \not\equiv V}{\{\mathrm{E}[M]\} \;\rightarrow\; \{\mathrm{E}[V]\}}$$

→ {(\c-> putChar c) *ch*}
→ {putChar *ch*}
$\xrightarrow{!ch}$ {return ()}

Treat primitive IO actions as "constructors"; so (putChar ch) is a value

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Semantics of Mutable State

With these basic tools in-hand, we can think about how to describe the semantics of other members of the "awkward squad."

Let's start with mutable state...

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Mutable variables in C

```
int x = 3;
x = x+1;
```

x is a location, initalised with 3

read x, add 1, store back into x

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Mutable variables in Haskell

```
do { x <- newIORef 3;
     v <- readIORef x;
     writeIORef x (v+1) }
```

x is a location, initialised with 3

read x

add 1, store back into x

```
newIORef   :: a -> IO (IORef a)
readIORef  :: IORef a -> IO a
writeIORef :: IORef a -> a -> IO ()
```

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Semantics for variables

Step 1: elaborate the program state

$$\begin{array}{llll} P,Q,R & ::= & \{M\} & \text{The main program} \\ & \mid & \langle M\rangle_r & \text{An } \texttt{IORef} \text{ named } r\text{, holding } M \\ & \mid & P \mid Q & \text{Parallel composition} \\ & \mid & \nu x.P & \text{Restriction} \end{array}$$

e.g. $\nu r,s.\ (\{M\} \mid \langle 3\rangle_r \mid \langle 89\rangle_s)$

Another IORef

Current set of names ("νr,s. ..." is shorthand for "νr. νs. ...")

An IORef named r, holding 3

The main program

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Semantics for variables

Live demo – evaluation of

```
do { x <- newIORef 3;
     v <- readIORef x;
     writeIORef x (v+1) }
```

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Semantics for variables

Step 2: add rules for reading, writing IORefs

$$\begin{array}{lcl} \{E[\texttt{readIORef}\ r]\}\ |\ \langle M\rangle_r & \rightarrow & \{E[\texttt{return}\ M]\}\ |\ \langle M\rangle_r \\ \{E[\texttt{writeIORef}\ r\ N]\}\ |\ \langle M\rangle_r & \rightarrow & \{E[\texttt{return}\ ()]\}\ |\ \langle N\rangle_r \end{array}$$

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Semantics for variables

Step 2: add rules for reading, writing IORefs

$$\begin{array}{lcl} \{E[\texttt{readIORef}\ r]\}\ |\ \langle M\rangle_r & \rightarrow & \{E[\texttt{return}\ M]\}\ |\ \langle M\rangle_r \\ \{E[\texttt{writeIORef}\ r\ N]\}\ |\ \langle M\rangle_r & \rightarrow & \{E[\texttt{return}\ ()]\}\ |\ \langle N\rangle_r \end{array}$$

But what if the main program is not "sitting next to" the relevant variable? We might need to rearrange the program state so that the rules above can apply...

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"Structural rules"

Intuition: A program state is a "soup" consisting of many IORefs and one main program

Step 3: add rules to bring "reagents" together

$$\begin{array}{rcl} P\ |\ Q & \equiv & Q\ |\ P \\ P\ |\ (Q\ |\ R) & \equiv & (P\ |\ Q)\ |\ R \end{array}$$

Stirring the soup

$$\frac{P \equiv P' \quad P' \xrightarrow{\alpha} Q' \quad Q' \equiv Q}{P \xrightarrow{\alpha} Q}\ (EQUIV)$$

Can look under "|"

$$\frac{P \xrightarrow{\alpha} Q}{P\ |\ R \xrightarrow{\alpha} Q\ |\ R}\ (PAR)$$

Can perform any transitions that could be performed on a stirred soup

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Restriction

Step 4: creation of fresh IORef names

$\{E[\texttt{newIORef}\ M]\} \rightarrow \{E[\texttt{return}\ ?]\}\ |\ \langle M\rangle_?$

What can we use as the IORef name???

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Restriction

Step 4: deal with fresh IORef names

Choose a name r that is not used already

$$\frac{r \notin fn\,(E, M)}{\{E[\texttt{newIORef}\ M]\} \rightarrow \nu r.(\{E[\texttt{return}\ r]\}\ |\ \langle M\rangle_r)}$$

Add r to the current set of names

Yield r as the result of newIORef

Put M in a new cell named r

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More "Structural rules"

Step 5: structural rules for restriction

Can float "ν" outwards (towards the top of the soup)

$$\begin{array}{rcll} \nu x.\nu y.P & \equiv & \nu y.\nu x.P & \\ (\nu x.P)\ |\ Q & \equiv & \nu x.(P\ |\ Q), & x \notin fn\,(Q) \\ \nu x.P & \equiv & \nu y.P[y/x], & y \notin fn\,(P) \end{array}$$

Can look under "ν"

$$\frac{P \xrightarrow{\alpha} Q}{\nu x.P \xrightarrow{\alpha} \nu x.Q}\ (NU)$$

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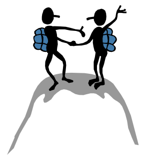
Phew!

Quite a lot of technical machinery!



But:

- It's standard, widely-used machinery (esp. in process calculi), so it's worth getting used to
- It scales to handle non-determinism and concurrency (as we will see next!)

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