A Gentle Introduction to Program Analysis

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January 21, 2014
Programming Languages Mentoring Workshop
What is Program Analysis?

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- Program analysis is about developing algorithms and tools that can analyze **other programs**
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Applications of Program Analysis

- **Bug finding.** e.g., expose as many assertion failures as possible
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- **Verification.** e.g., does the program always behave according to its specification?

- **Compiler optimizations.** e.g., which variables should be kept in registers for fastest memory access?

- **Automatic parallelization.** e.g., is it safe to execute different loop iterations on parallel?
Dynamic vs. Static Program Analysis

- Two flavors of program analysis:
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- **Dynamic analysis**: Analyzes program while it is running
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- **Static analysis**: Analyzes source code of the program
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**Static**
- + reasons about all executions
- - less precise

**Dynamic**
- + more precise
- - results limited to observed executions
Typical static analysis question: "Given source code of program P and desired property Q, does P exhibit Q in all possible executions?"
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Static Analysis

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- **Unsound**: May say program is safe even though it is unsafe
- **Sound, but incomplete**: May say program is unsafe even though it is safe
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Many static analysis techniques are sound but incomplete.
Key idea: Overapproximate (i.e., abstract) program behavior
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- Bad states outside over-approximation
  \[ \Rightarrow \text{Program safe} \]

- Bad states inside over-approximation, but outside $P$
  \[ \Rightarrow \text{false alarm} \]
How to design sound static analyses?

Key idea: Overapproximate (i.e., abstract) program behavior

- Bad states outside over-approximation ⇒ Program safe

- Bad states inside over-approximation, but outside $P$ ⇒ false alarm

⇒ Goal: Construct abstractions that are precise enough (i.e., few false alarms) and that scale to real programs
Examples of Abstractions

There is no "one size fits all" abstraction. What information is useful depends on what you want to prove about the program!

Application

Useful abstraction

No division-by-zero errors

zero vs. non-zero

Data structure verification

list, tree, graph, . . .

No out-of-bounds array accesses

ranges of integer variables
There is no “one size fits all” abstraction

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Examples of Abstractions

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How to Create Sound Abstractions?

Useful theory for understanding how to design sound static analyses is abstract interpretation. Seminal '77 paper by Patrick & Radhia Cousot.

Not a specific analysis, but rather a framework for designing sound-by-construction static analyses.

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First Step: Design An Abstract Domain

- An **abstract domain** is just a set of **abstract values** we want to track in our analysis.
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- For our example, let’s fix the following abstract domain:
  - **pos**: \( \{ x \mid x \in \mathbb{Z} \land x > 0 \} \)
  - **zero**: \( \{ 0 \} \)
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  - \( \top \): “Don’t know”, represents any value
  - \( \bot \): Represents empty-set
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- In addition, every abstract domain contains:
  - \( \top \) (top): “Don’t know”, represents any value
  - \( \bot \) (bottom): Represents empty-set
Abstraction function ($\alpha$) maps sets of concrete elements to the most precise value in the abstract domain.
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Second step: Abstraction and concretization functions

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- **Concretization function** ($\gamma$) maps each abstract value to sets of concrete elements
  - $\gamma(\text{pos}) = \{ x \mid x \in \mathbb{Z} \land x > 0 \}$
Concretization function defines partial order on abstract values:

Furthermore, in an abstract domain, every pair of elements has a lub and glb ⇒ mathematical lattice.
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\[ A_1 \leq A_2 \text{ iff } \gamma(A_1) \subseteq \gamma(A_2) \]
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Least upper bound of two elements is called their join – useful for reasoning about control flow in programs
Important property of the abstraction and concretization function is that they are almost inverses:
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C \subseteq \gamma(\alpha(C))
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This is called a \textit{Galois insertion} and captures the soundness of the abstraction.
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Step 3: Abstract Semantics

- Given abstract domain, $\alpha, \gamma$, need to define abstract transformers (i.e., semantics) for each statement.
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**Operational Semantics**

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S: Var → Concrete value

x = y op z

S': Var → Concrete value
```
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Operational Semantics

- $S$: Var $\rightarrow$ Concrete value
- $S'$: Var $\rightarrow$ Concrete value
- $x = y \text{ op } z$

Abstract Semantics

- $A$: Var $\rightarrow$ Abstract value
- $A'$: Var $\rightarrow$ Abstract value
- $x = y \text{ op } z$
For our sign analysis, we can define abstract transformer for \( x = y + z \) as follows:

<table>
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<tr>
<th></th>
<th>pos</th>
<th>neg</th>
<th>zero</th>
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<th>⊤</th>
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For our sign analysis, we can define abstract transformer for $x = y + z$ as follows:

|       | pos | neg | zero | non-neg | $\top$ | $\bot$
|-------|-----|-----|------|---------|-------|-------
| pos   | pos | $\top$ | pos | pos     | $\top$ | $\bot$
| neg   | $\top$ | neg | neg | $\top$ | $\top$ | $\bot$
| zero  | pos | neg | zero | non-neg | $\top$ | $\bot$
| non-neg | pos | $\top$ | non-neg | non-neg | $\top$ | $\bot$
| $\top$ | $\top$ | $\top$ | $\top$ | $\top$ | $\top$ | $\bot$
| $\bot$ | $\bot$ | $\bot$ | $\bot$ | $\bot$ | $\bot$ | $\bot$

To ensure soundness of static analysis, must prove that abstract semantics faithfully models concrete semantics.
Putting It All Together

Fixed-point engine
Abstract domain
Abstract semantics

P
Putting It All Together

Fixed-point

Abstract domain

Abstract semantics

Fixed-point engine

P
**Fixed-point computation:** Repeated symbolic execution of the program using abstract semantics until our approximation of the program reaches an equilibrium.
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**Least fixed-point:** Start with underapproximation and grow the approximation until it stops growing.
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**Least fixed-point:** Start with underapproximation and grow the approximation until it stops growing.

Assuming correctness of your abstract semantics, the least fixed point is an overapproximation of the program!
Represent program as a **control-flow graph**
Performing Least Fixed Point Computation

- Represent program as a **control-flow graph**
- Want to compute abstract values at every program point
Performing Least Fixed Point Computation

- Represent program as a **control-flow graph**
- Want to compute abstract values at every program point
- Initialize all abstract states to $\bot$

```
x = 0
y = 1
```

```
loop head

y <= n
```

```
x = x+1
x = x + y
```

```
exit block
```

```
branch
z = 0
z != 0
```

```
x = x+1
x = x + y
```

```
loop end
y = y + 1
```

```
```
Performing Least Fixed Point Computation

- Represent program as a **control-flow graph**
- Want to compute abstract values at every program point
- Initialize all abstract states to ⊥
- Repeat until no abstract state changes at any program point:
Performing Least Fixed Point Computation

- Represent program as a **control-flow graph**

- Want to compute abstract values at every program point

- Initialize all abstract states to $\perp$

- Repeat until no abstract state changes at any program point:
  - Compute abstract state on entry to a basic block $B$ by taking the **join** of $B$’s predecessors
Performing Least Fixed Point Computation

- Represent program as a **control-flow graph**
- Want to compute abstract values at every program point
- Initialize all abstract states to \( \bot \)
- Repeat until no abstract state changes at any program point:
  - Compute abstract state on entry to a basic block \( B \) by taking the **join** of \( B \)'s predecessors
  - Symbolically execute each basic block using abstract semantics
```c
x = 0;
y = 0;
while(y <= n) {
    if (z == 0) {
        x = x + 1;
    } else {
        x = x + y;
    }
    y = y + 1;
}
```

Diagram:
- **loop head**
  - **x = 0**
  - **y = 1**
- **exit block**
  - **y <= n**
- **branch**
  - **z = 0**
  - **z != 0**
- **loop end**
  - **x = x + 1**
  - **x = x + y**
  - **y = y + 1**
An Example

Is $x$ always non-negative inside the loop?

```plaintext
x = 0;
y = 0;
while(y <= n) {
   if (z == 0) {
      x = x+1;
   } else {
      x = x + y;
   }
   y = y+1
}
```
Fixed-Point Computation

```
x = 0
y = 1
```

```
loop head

exit block
branch

y <= n
z = 0
z != 0

x = x + 1
x = x + y

loop end
y = y + 1
```
Fixed-Point Computation

\[
x = 0, y = 1
\]

\[
x = x+1, y = x+y
\]

\[
y \leq n
\]

\[
z = 0, z \neq 0
\]

\[
y = y+1
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Fixed-Point Computation

\[ x = \bot, y = \bot \]
\[ x = z, y = \bot \]
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\[ x = 0, y = 1 \]

loop head

exit block

branch

\[ y \leq n \]
\[ z = 0 \]
\[ z \neq 0 \]

loop end

\[ x = x + 1 \]
\[ x = x + y \]

\[ y = y + 1 \]

\[ x = \bot, y = \bot \]
Fixed-Point Computation

\[
x = 0
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\[
y = 1
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x = Z, y = \bot
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x = Z, y = P
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loop end
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x = Z, y = P
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Fixed-Point Computation

$x = \perp, y = \perp$
$x = Z, y = \perp$
$x = Z, y = P$

$x = 0$
y = 1

loop head

$y \leq n$

z = 0
z $\neq$ 0

x = Z, y = P

loop end

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x = Z, y = P
Fixed-Point Computation

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Exit block

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Branch

\[ x = x + 1 \]
\[ x = x + y \]
\[ x = Z, y = P \]

Loop end

\[ y = y + 1 \]

\[ x = \bot, y = \bot \]
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Fixed-Point Computation

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x = 0, \ y = 1
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Fixed-Point Computation

\[ x = 0 \]
\[ y = 1 \]

**loop head**

- \( x = Z, y = P \)
- \( x = Z, y = P \)

**exit block**

- \( x = Z, y = P \)

**branch**

- \( x = Z, y = P \)
- \( x = Z, y = P \)
- \( x = Z, y = P \)
- \( x = Z, y = P \)
- \( x = P, y = P \)
- \( x = P, y = P \)
- \( x = P, y = P \)

**loop end**

- \( x = P, y = P \)
- \( x = P, y = P \)
- \( x = P, y = P \)

- \( y = y + 1 \)
Fixed-Point Computation

- $x = \perp, y = \perp$
- $x = Z, y = \perp$
- $x = Z, y = P$
- $x = 0, y = 1$
- $x = x + 1, x = x + y$
- $x = P, y = P$
- $x = P, y = P$
- $x = P, y = P$
- $y \leq n$
- $z = 0$
- $z \neq 0$
- $x = Z, y = P$
- $y = y + 1$
- $x = P, y = P$
- $x = P, y = P$
- $x = P, y = P$
Fixed-Point Computation

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In this example, we quickly reached least fixed point – but does this computation always terminate?
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Yes, assuming abstract domain forms complete lattice
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- Yes, assuming abstract domain forms complete lattice
- This means every subset of elements (including infinite subsets) have a LUB
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- Yes, assuming abstract domain forms complete lattice

- This means every subset of elements (including infinite subsets) have a LUB

- Unfortunately, many interesting domains do not have this property, so we need widening operators for convergence.
Considered only one static analysis approach, but illustrates two key ideas underlying program analysis:

- **Abstraction:** Only reason about certain properties of interest.
- **Fixed-point computation:** Allows us to obtain sound overapproximation of the program.

But many static analyses also differ in several ways:

- **Flow (in)sensitivity:** Some analyses only compute facts for the whole program, not for every program point.
- **Path sensitivity:** More precise analyses compute different facts for different program paths.
- **Analysis direction:** Forwards vs. backwards.
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But many static analyses also differ in several ways:
Lessons To Take Away

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Many open problems in program analysis
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- Precise and scalable heap reasoning
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- Concurrency
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Many open problems in program analysis

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Exciting area with lots of interesting topics to work on!
Many open problems in program analysis

- Precise and scalable heap reasoning
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- ...
Challenges and Open Problems

Many open problems in program analysis

- Precise and scalable heap reasoning
- Concurrency
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- ...

Exciting area with lots of interesting topics to work on!
If you are interested in program analysis or verification, consider applying to UT Austin!