

CIS 673: Computer-Aided Verification
Homework 2: Due October 25

1. Consider the boolean expression

$$(x_1 \wedge x_2 \wedge x_3) \vee (\neg x_2 \wedge x_4) \vee (\neg x_3 \wedge x_4)$$

Choose a variable ordering for the variables $\{x_1, x_2, x_3, x_4\}$, and draw the resulting BDD. Can you reduce the size of the BDD by reordering the variables?

2. Given a BDD B over a set X of variables, a variable x , and a boolean value $m \in \{true, false\}$, give an algorithm to construct the BDD representing the function $r(B)[x := m]$. That is, output BDD should be obtained by simplifying the input BDD by replacing the variable x with the constant m . What is the running time of your algorithm in terms of the number of vertices of the input B .
3. Write an STL formula that specifies the equal-opportunity requirement for mutual exclusion protocols: if process P_1 attempts to enter the critical section when process P_2 is in its noncritical section, then P_2 may enter its critical section at most once before P_1 is allowed to enter its critical section.
4. Prove that stutter-closed bisimilarity \cong^B is an abstract semantics for Stl^U .
5. The logic STL^+ has the syntax

$$\phi ::= p \mid \phi \vee \psi \mid \neg \phi \mid \exists \bigcirc \phi \mid \phi \exists \mathcal{U}^+ \psi,$$

where the semantics of the strict-until operator is defined by the clause

$$s \models_K \psi \exists \mathcal{U}^+ \phi \quad \text{iff} \quad \begin{array}{l} \text{there is a source-}s \text{ trajectory } \bar{s}_{0..m} \text{ of } K \text{ such that (1) } m > 0 \\ \text{(2) } s_m \models_K \phi \text{ and} \\ \text{(3) for all } 0 \leq i \leq m, s_i \models_K \phi \vee \psi. \end{array}$$

Thus, while $p \exists \mathcal{U} q$ can be satisfied in a state s by satisfying q in s , satisfaction of $p \exists \mathcal{U}^+ q$ in a state s requires a source- s trajectory that is of at least length 2 and leads to a state satisfying q . Is the logic STL as expressive as the logic STL^+ ? Is the logic STL^+ as expressive as the logic STL?