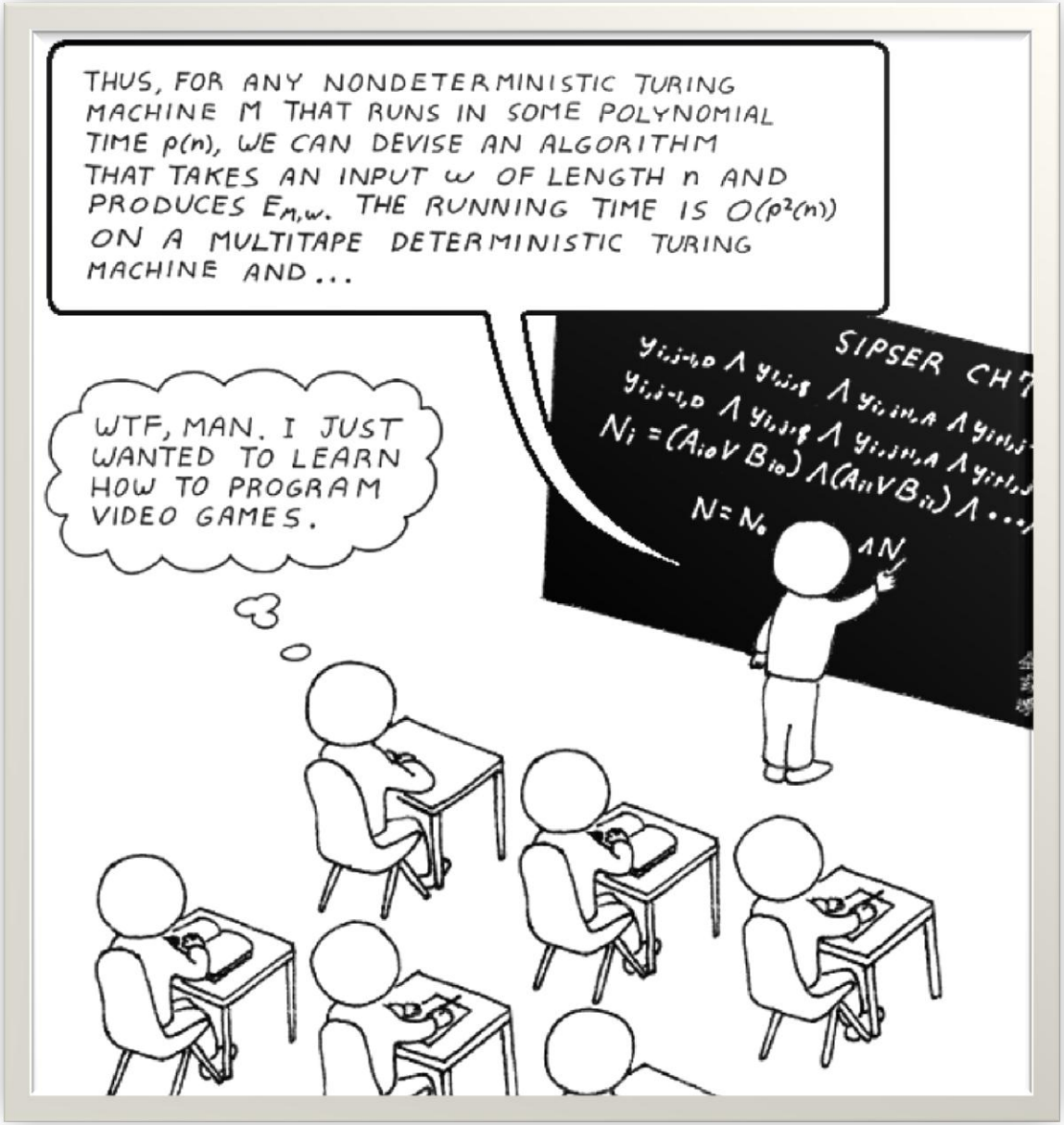
ALGORITHMIC GAME THEORY

Incentive and Computation


THUS, FOR ANY NONDETERMINISTIC TURING MACHINE M THAT RUNS IN SOME POLYNOMIAL TIME p(n), WE CAN DEVISE AN ALGORITHM THAT TAKES AN INPUT w OF LENGTH n AND PRODUCES $E_{M,w}$. THE RUNNING TIME IS $O(p^2(n))$ ON A MULTITAPE DETERMINISTIC TURING MACHINE AND...
SIPSER CH
$N_i = (A_{i,0} \vee B_{i,0}) \wedge (A_{i,1} \vee B_{i,1}) \wedge \ldots$
$N = N_0$
$\wedge N$
WTF, MAN. I JUST WANTED TO LEARN HOW TO PROGRAM VIDEO GAMES.

Basic Parameters

When: Tuesday/Thursday, 1:30-2:50

Where: Here!

Who: Professor Aaron Roth

How: 3-4 problem sets (40%), 2 exams (50%),
Attendance + Participation (10%)

Participation includes questions and discussion on *Piazza*. Make an account.

Basic Parameters

Homework Policy: Reasonable Person Principle.
Feel free to work together.

(List who you worked with)

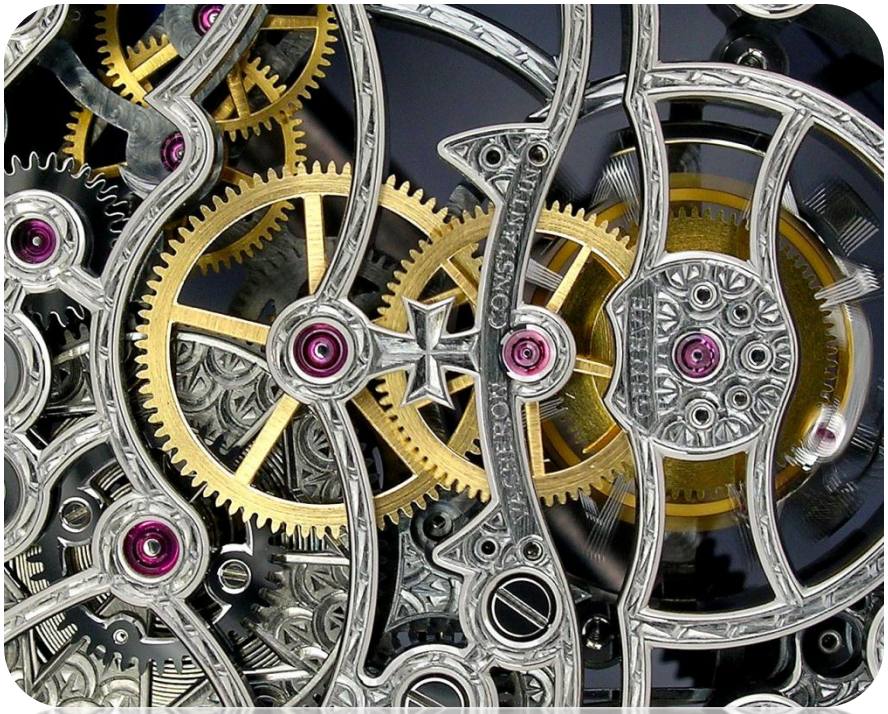
(Everyone turns in their own assignment)

(5% off per day late. One “free” day.)

Algorithm Design
vs.
Mechanism Design

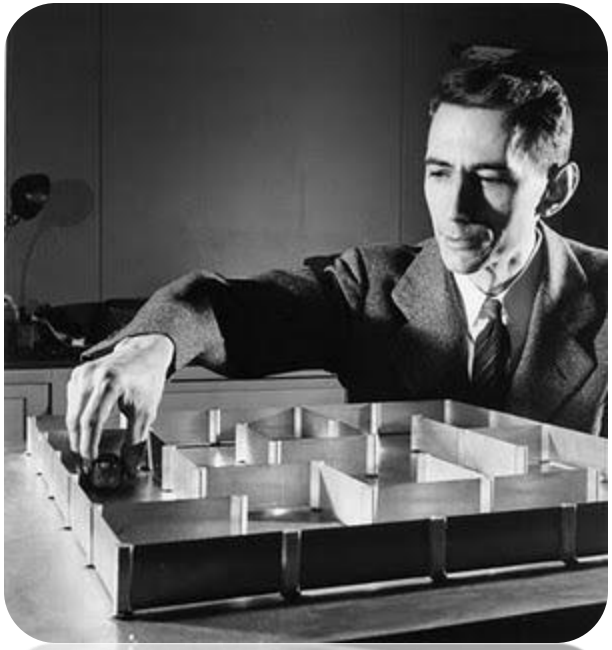
Algorithms vs. Games

- If we control the whole system, we can just design an algorithm.



Algorithms vs. Games

- Otherwise, we have to design the *constraints* and *incentives* so that agents in the system work to achieve our goals.



Algorithms vs. Games

- And once the rules are in place, predict what will happen…

This comes up all the time in CS…

This Class

Game Theory Basics

What is a game?

A set of Players

A set of Actions

A set of Payoffs

Game Theory

			
	0,0	-1,1	1,-1
	1,-1	0,0	-1 , 1
	-1,1	1 , -1	0,0

How should you play a game?

Myopically play to maximize your payoff?

But other players will respond…

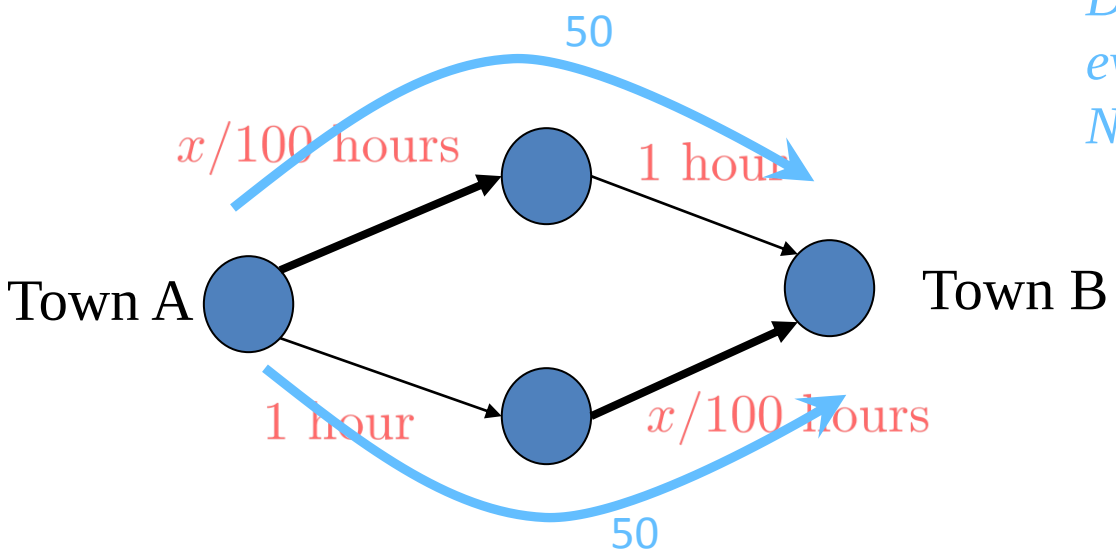
Can we predict outcomes?

What happens if everyone plays “Rationally”?

What if some people don’t?

Traffic Routing

Delay is 1.5 hours for everybody at the unique Nash equilibrium



Suppose 100 drivers leave from town A towards town B.

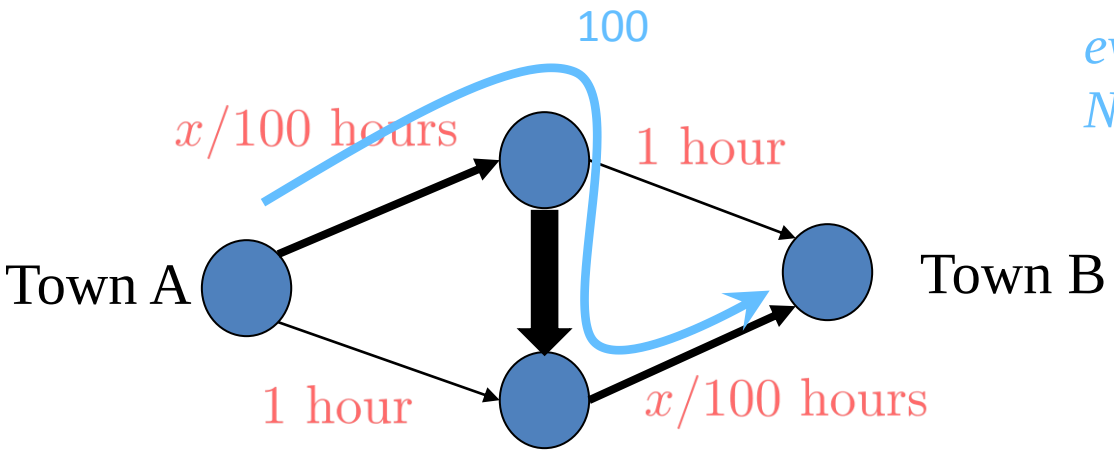
Every driver wants to minimize her own travel time.

What is the traffic on the network?

In any unbalanced traffic pattern, all drivers on the most loaded path have incentive to switch their path.

Traffic Routing

Delay is 2 hours for everybody at the unique Nash equilibrium

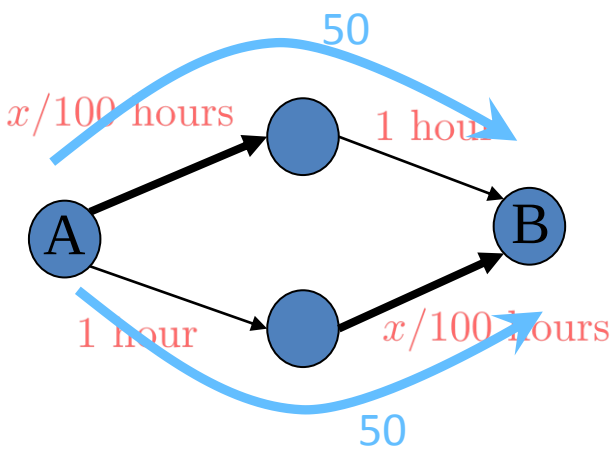


A benevolent governor builds a superhighway connecting the short roads of the network.

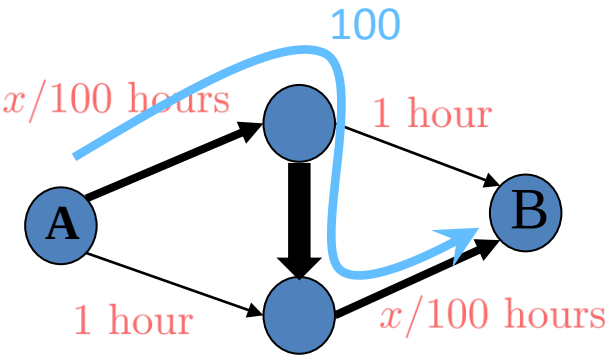
What is the traffic on the network now?

No matter what the other drivers are doing it is always better for me to follow the zig-zag path.

Traffic Routing



vs



Adding a fast road on a road-network is not always a good idea!

Braess's paradox

4/3

In the RHS network there exists a traffic pattern where all players have delay 1.5 hours.

$$\text{PoA} = \frac{\text{performance of system in worst Nash equilibrium}}{\text{optimal performance if drivers did not decide on their own}}$$

Price of Anarchy: measures the loss in system performance due to free-will

Can we influence outcomes?

Can we change the rules of the game to encourage “rational” players to do what we want?

Computation?

How does computational complexity affect our predictions?

How does computational complexity limit our design choices?

Information?

How does limited information affect our predictions?

How does limited information limit our design choices?

How well can we expect to do?

When playing a prediction game?

(Horse betting)

(Investing in stocks)

How Small Changes in Rules Lead to Behavioral Changes

Dollar Auction

First Price Sealed Bid

- Rules:
  - Everyone write down your name and bid on a piece of paper, and keep it secret.
  - I'll collect the bids.
  - The highest bidder gets the $5, and pays the amount of their bid.
  - No Talking!

English Auction

- Rules:
  - Price begins at $0 and raises in $0.25 increments
  - Raise your hand to bid.
  - When bidding stops, highest bidder wins, pays their bid, gets $5
  - No collusion.

2-Pay Auction

- Rules:
  - Price begins at $0 and raises in $0.25 increments
  - Raise your hand to bid.
  - When bidding stops, highest bidder wins, pays their bid, gets $5
  - 2nd highest bidder also pays their bid, gets nothing.
  - No collusion.

What Happened?

First Price Auction

Incentive to bid *below* $5

English Auction

Incentive to stay in until $5

2-Pay Auction

Strong incentive not to be 2nd bidder.

Guess Two-Thirds of the Average

- k players $p_1, p_2, p_3, \ldots, p_k$

- each player submits a number in [0,100]

$$x_1, x_2, \ldots, x_k$$

- compute

$$\bar{x} := \frac{1}{k} \sum_{i=1}^{k} x_i$$

Let's Play!

- find x_j, closest to $\frac{2}{3}\bar{x}$

- player p_j wins \$100, all other players win nothing

Guess Two-Thirds of the Average

Is it rational to play above $\frac{2}{3} \cdot 100$?

A: no (why?)

Given that no rational player will play above $\frac{2}{3} \cdot 100$ is it rational to play above $(2/3)^2 \cdot 100$?

A: no (same reasons)

⋮

All rational players should play 0.

The all-zero strategy is the only Nash equilibrium of this game.

Rationality versus common knowledge of rationality

historical facts: 21.6 was the winning value in a large internet-based competition organized by the Danish newspaper Politiken. This included 19,196 people and with a prize of 5000 Danish kroner.

IS COMMON KNOWLEDGE REASONABLE? THE PARABLE OF THE ISLANDERS

On an isolated island, 100 logicians live in total isolation. They have developed a quirky culture:

1. If any islander can deduce that he has blue eyes, he must kill himself on the beach of midnight that night.
2. No islander may tell another that she has blue eyes.

Another quirk: All of the islanders are blue-eyed. Since there are no mirrors and the water is murky, no one has ever known their own eye color, and they have lived in harmony for hundreds of years with no suicides.

IS COMMON KNOWLEDGE REASONABLE? THE PARABLE OF THE ISLANDERS

One day, an explorer arrives on the island and addresses the islanders with a faux-pas.

"At Least One of You Has Blue Eyes," he tells them.

Due to the explorers terrible breach in manners, the islanders quickly dispatch of him, but the damage has already been done.

Of course – the explorer told the islanders nothing new. Each islander could see for himself that there were at least 99 islanders with blue eyes. And yet…

Homework: Discuss on Piazza -- Has anything changed? What happens?