Algorithmic Game Theory: Problem Set 6
Due on Tuesday, April 18
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Remember you can work together on problem sets, but list everyone you worked with, and everyone turn in their own assignment.

Efficient Implementation of VCG (15 pts)
Consider an auction setting in which the auctioneer is selling $m$ identical copies of some good. Buyers can be assigned multiple copies of the good, and have valuation functions $v_i$ which map the number of copies of the good they receive to their total value: $v_i : \{0, 1, \ldots, m\} \to \mathbb{R}$. Write $v_{i,j}$ for buyer $i$'s marginal value for the $j$'th copy of a good – i.e. let $v_{i,j}$ be such that we can write $v_i(k) = \sum_{j=1}^{k} v_{i,j}$. Suppose further that each buyer $i$ has decreasing marginal utility for these goods – in other words, $v_{i,1} \geq v_{i,2} \geq \ldots \geq v_{i,m}$. For simplicity you can assume that all $v_{i,j}$ are distinct.

1. Give a simple greedy algorithm for implementing the allocation rule of the VCG mechanism in this setting. Prove that your algorithm optimizes welfare. Does this algorithm still work if buyer valuations do not satisfy the decreasing marginal utility property? (10 pts)

2. Give a simple description of the payment of a bidder in the VCG mechanism as a sum of marginal valuations reported by the other bidders. (5 pts)

Revenue Extractors and the VCG Mechanism (30 pts)
In this problem, we will consider a single-parameter auction setting in which the auctioneer has $m$ identical units of a good, but each agent only wants one copy of the good. The set of feasible alternatives is $A = \{S \subset \{1, \ldots, n\} : |S| \leq m\}$, and buyer $i$ obtains value $v_i$ for alternative $S$ if $i \in S$ and 0 otherwise. We will think about the behavior of the profit extractor auction with revenue target $R$, and compare that to the behavior of the VCG mechanism.

1. Prove that whenever the VCG mechanism obtains revenue at least $R$ in this setting, so does the profit extractor. (10 pts)

2. Show that there exists a valuation profile for which the profit extractor obtains revenue $R$, but the VCG mechanism obtains revenue less than $R$. (5 pts)

3. Show that the profit extractor is group strategyproof, meaning that no set $C$ of bidders can coordinate on a set of false bids in a way that strictly increases the utility of at least one member of $C$ without strictly decreasing the utility of some other member (10 pts).

4. Is the VCG mechanism group strategyproof in this setting? Prove your answer. (5 pts)
Knapsack Auctions (15 pts)

1. In class, we showed a 2-approximation algorithm for the knapsack auction, and proved that it was a monotone allocation rule. Give an explicit description of the payment rule which makes it a dominant strategy to report valuations truthfully. (5 pts)

2. Now consider a variant of the knapsack auction in which each bidder $i$ has to report both his value $v_i$ and his size $w_i$. This is no longer a single parameter domain. An allocation rule $x(v', w')$ now specifies the amount of capacity allocated to each bidder as a function of their reported bids and sizes. The utility of buyer $i$ is defined to be $v_i - p_i(v', w')$ if she gets her full capacity – i.e. if $x_i(v', w') \geq w_i$, and to be $-p_i(v, w)$ otherwise (i.e. she gets no value for getting capacity less than her full size). Consider the 2-approximation we considered in class that simply takes the reported sizes $w'$ at face value (and define $x_i(v', w') = w'_i$ if the algorithm allocates buyer $i$ with these reports, and $x_i(v', w') = 0$ otherwise), and uses the same payment rule computed above. Does this mechanism make reporting true values and sizes a dominant strategy? Prove it if so, or give an explicit counter-example. (10 pts)