Tutorial
Graph Based Image Segmentation

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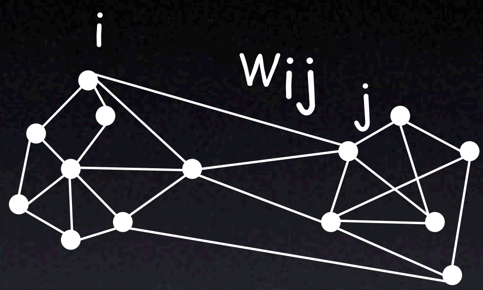
Topics

- Computing segmentation with graph cuts
- Segmentation benchmark, evaluation criteria
- Image segmentation cues, and combination
- Muti-grid computation, and cue aggregation



$$\mathbf{G} = \{\mathbf{V}, \mathbf{E}\}$$

V: graph nodes

E: edges connection nodes

↔

Image = { pixels }

Pixel similarity

Topics

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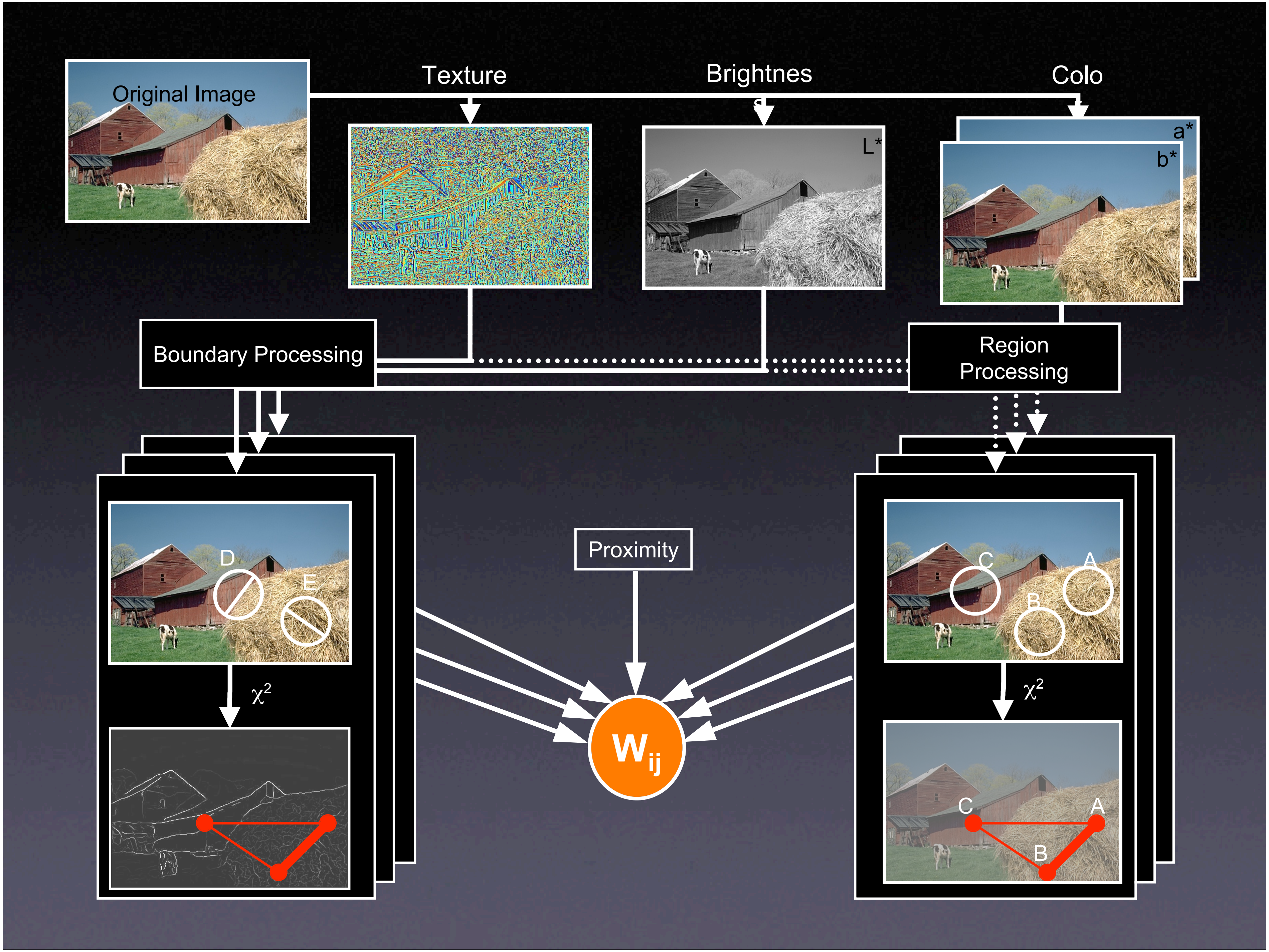

Original Image
Texture
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Boundary Processing
Region
Processing
Proximity
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Topics

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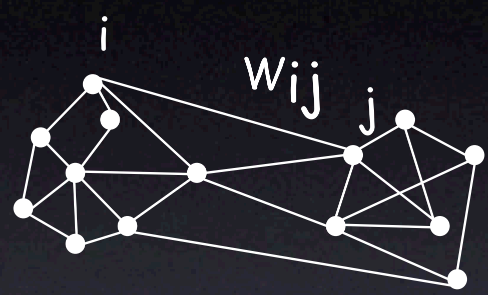
Part I: Graph and Images

Jianbo Shi

Graph Based Image Segmentation



$$\mathbf{G} = \{\mathbf{V}, \mathbf{E}\}$$

V: graph nodes

E: edges connection nodes

Image = { pixels }

Pixel similarity

Segmentation = Graph partition

Right partition cost function?

Efficient optimization algorithm?

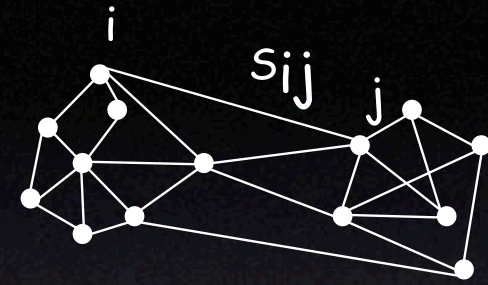
Graph Terminology

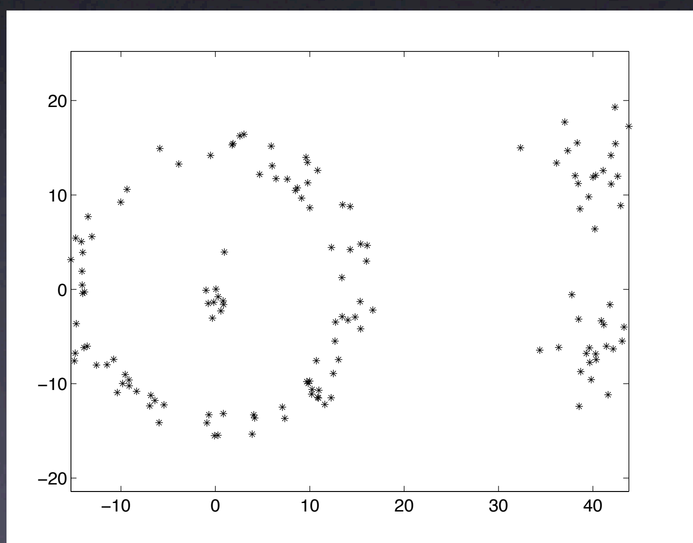
adjacency matrix,
degree,
volume,
graph cuts

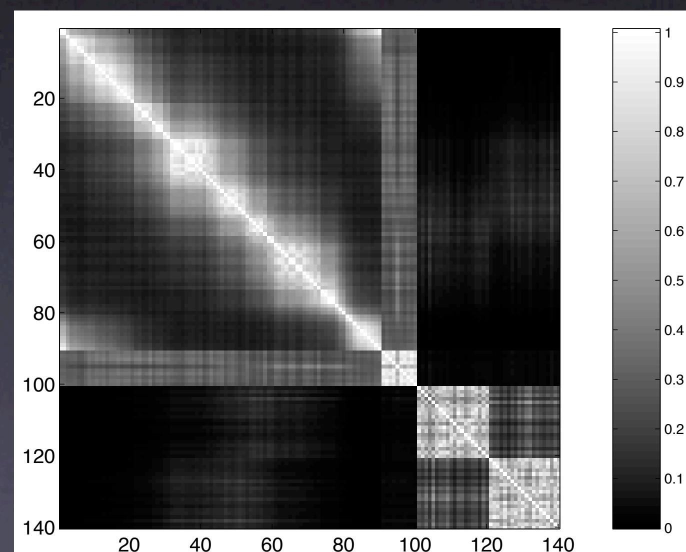
Graph Terminology

Similarity matrix $S = [\, s_{ij} \,]$

is generalized adjacency matrix

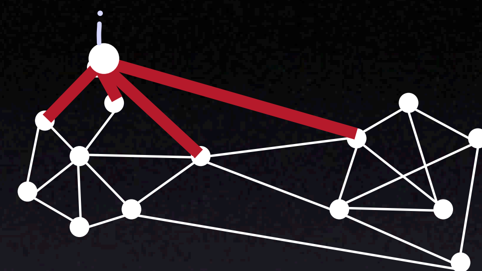


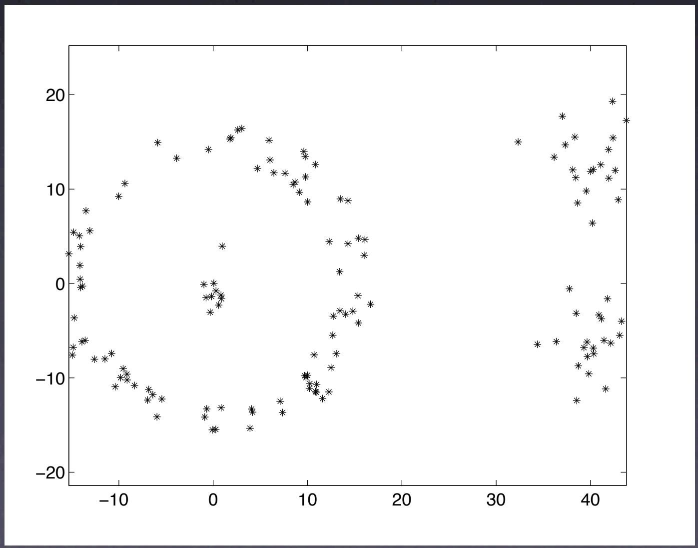


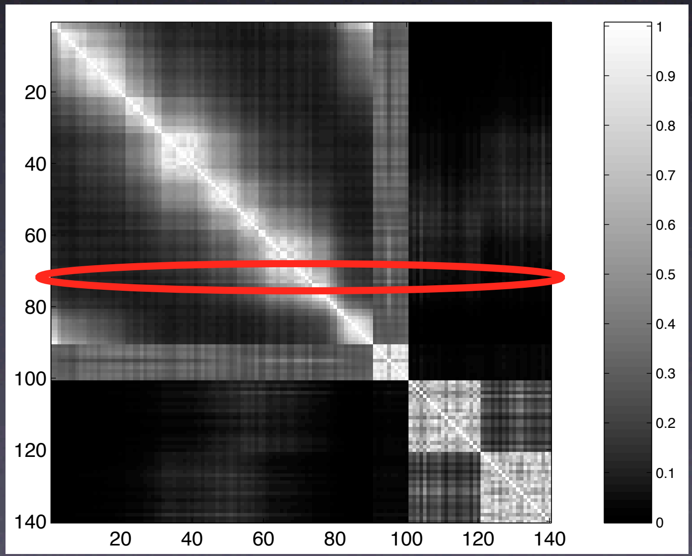


Graph Terminology

Degree of node: $d_i = \sum_j S_{ij}$



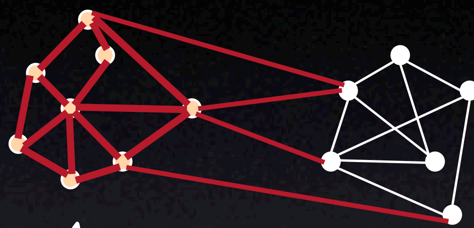


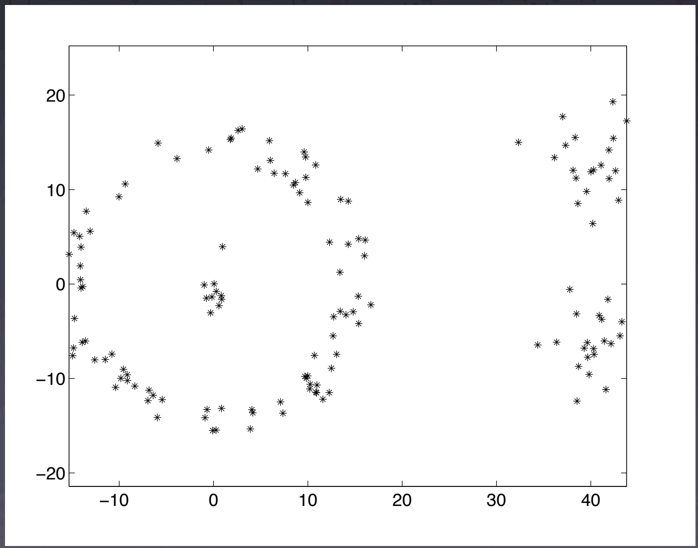


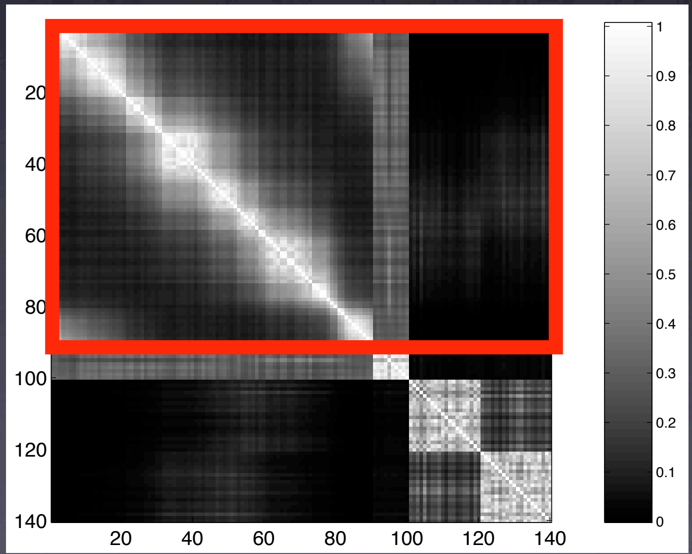
Graph Terminology

Volume of set:

$$vol(A) = \sum_{i \in A} d_i,\ A \subseteq V$$

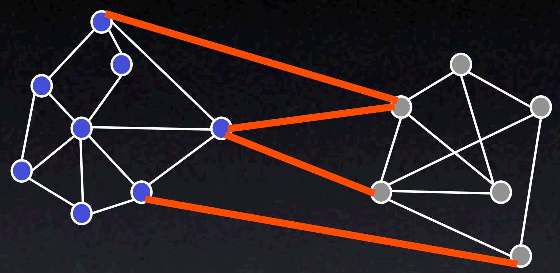


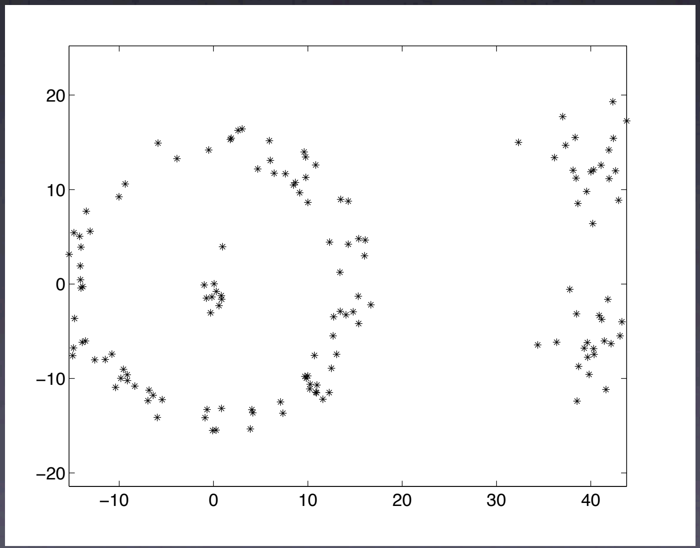


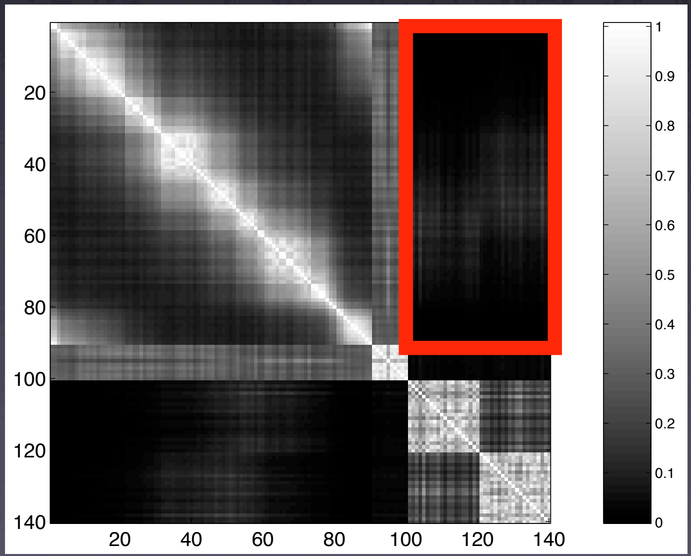


Cuts in a graph

$$cut(A, \bar{A}) = \sum_{i \in A, j \in \bar{A}} S_{i,j}$$

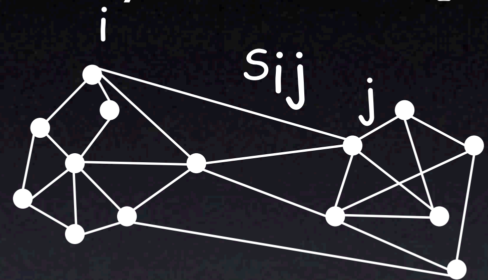




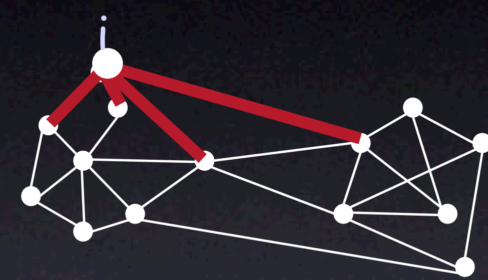


Graph Terminology

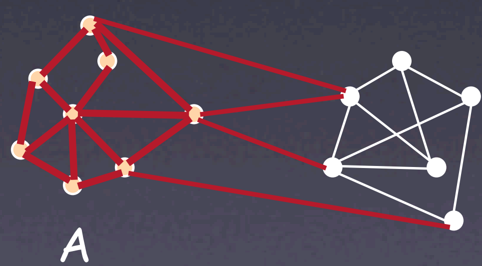
Similarity matrix $S = [\ S_{ij}\ ]$



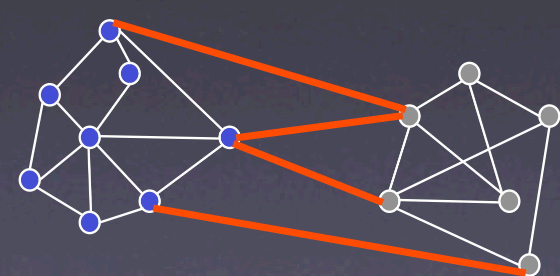
Degree of node: $d_i = \sum_j S_{ij}$



Volume of set:



Graph Cuts

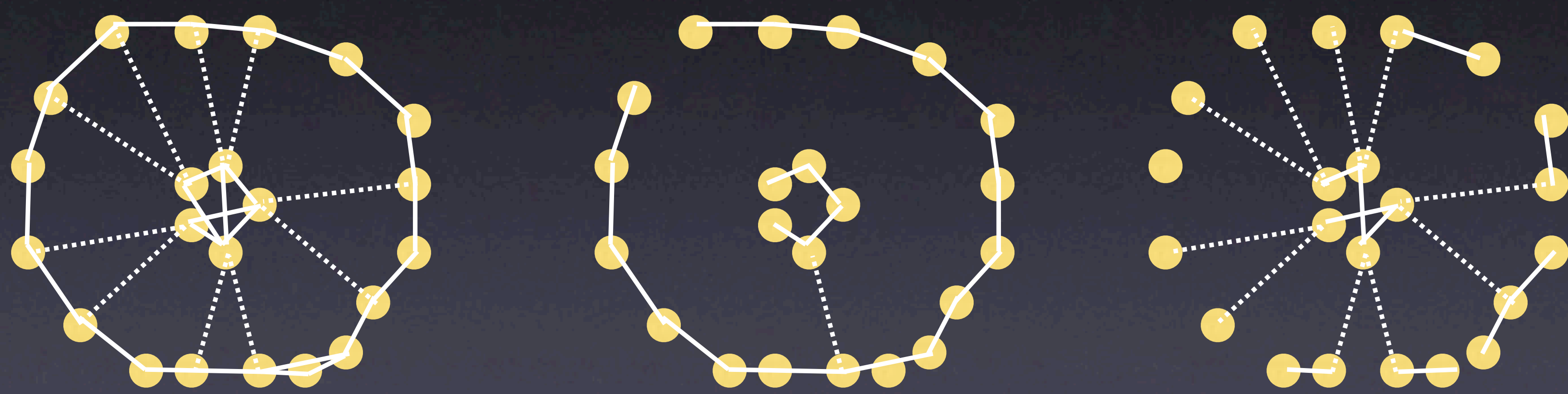


Useful Graph Algorithms

- Minimal Spanning Tree
- Shortest path
- s-t Max. graph flow, Min. cut

Minimal/Maximal Spanning Tree

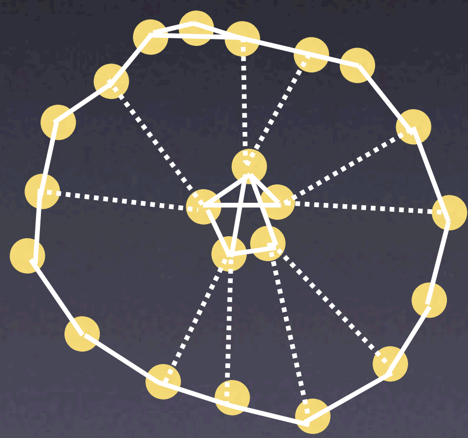
Tree is a graph G without cycle

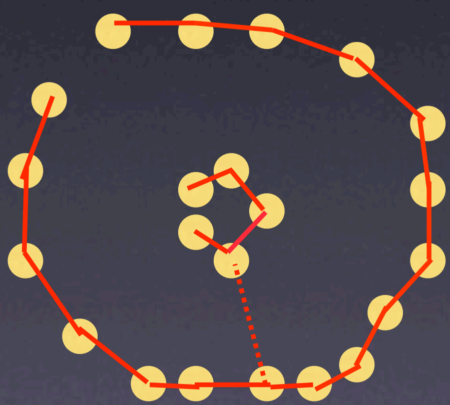


Graph Maximal Minimal

Kruskal’s algorithm

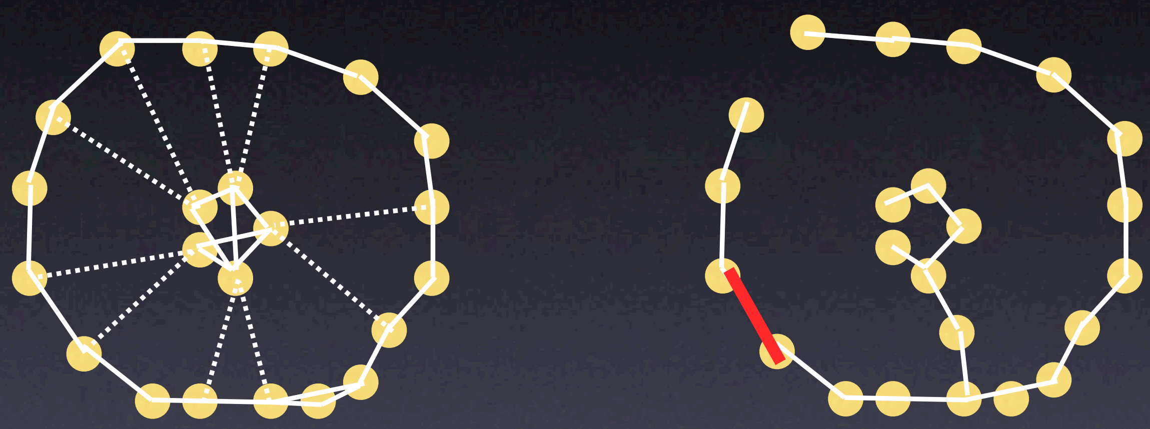
- sort the edges of G in increasing order by length
- for each edge e in sorted order
  if the endpoints of e are disconnected in S
  add e to S





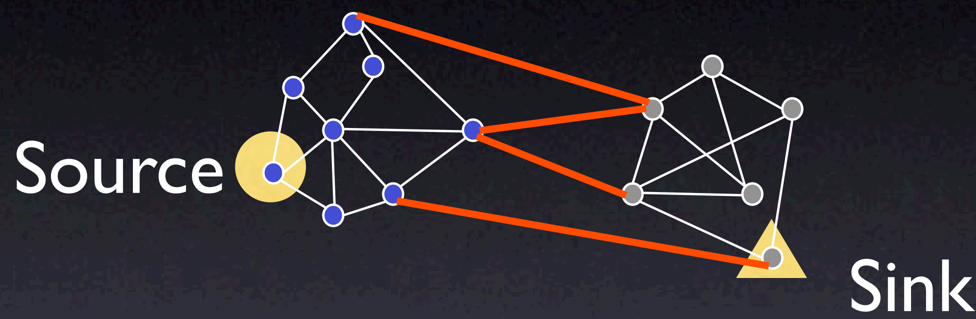
Randonmalized version can compute Typical cuts

Leakage problem in MST



Leakage

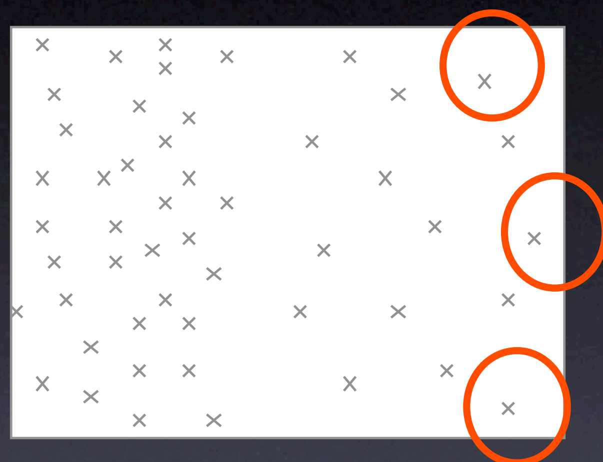
Graph Cut and Flow



1) Given a source (s) and a sink node (t)
2) Define Capacity on each edge, C_ij = W_ij
3) Find the maximum flow from s->t, satisfying the capacity constraints

Min. Cut = Max. Flow

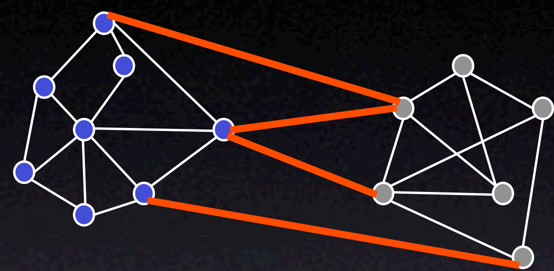
Problem with min cuts



Min. cuts favors isolated clusters

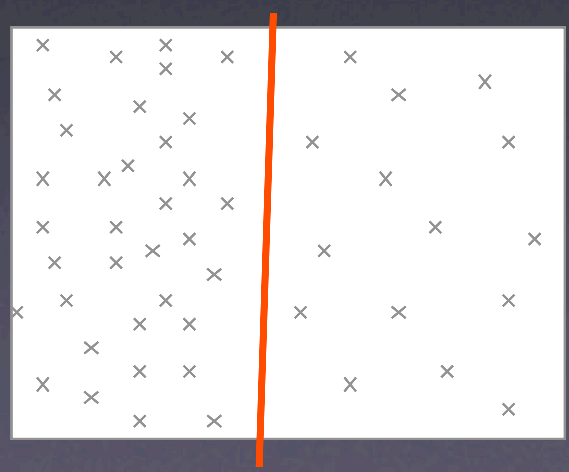
Normalize cuts in a graph

- (edge) Ncut = balanced cut



$$Ncut(A,B) = cut(A,B)\left(\frac{1}{vol(A)} + \frac{1}{vol(B)}\right)$$

NP-Hard!



Representation

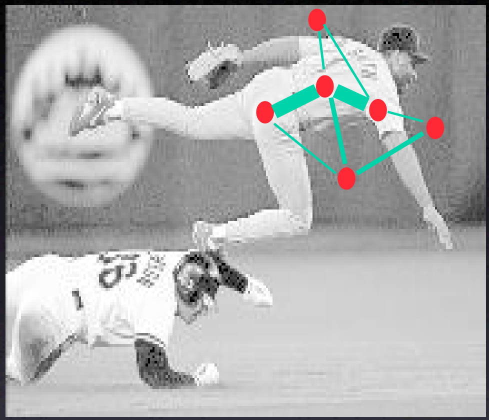
Partition matrix:

$$X = [X_1, ..., X_K]$$

segments

$$X = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

pixels

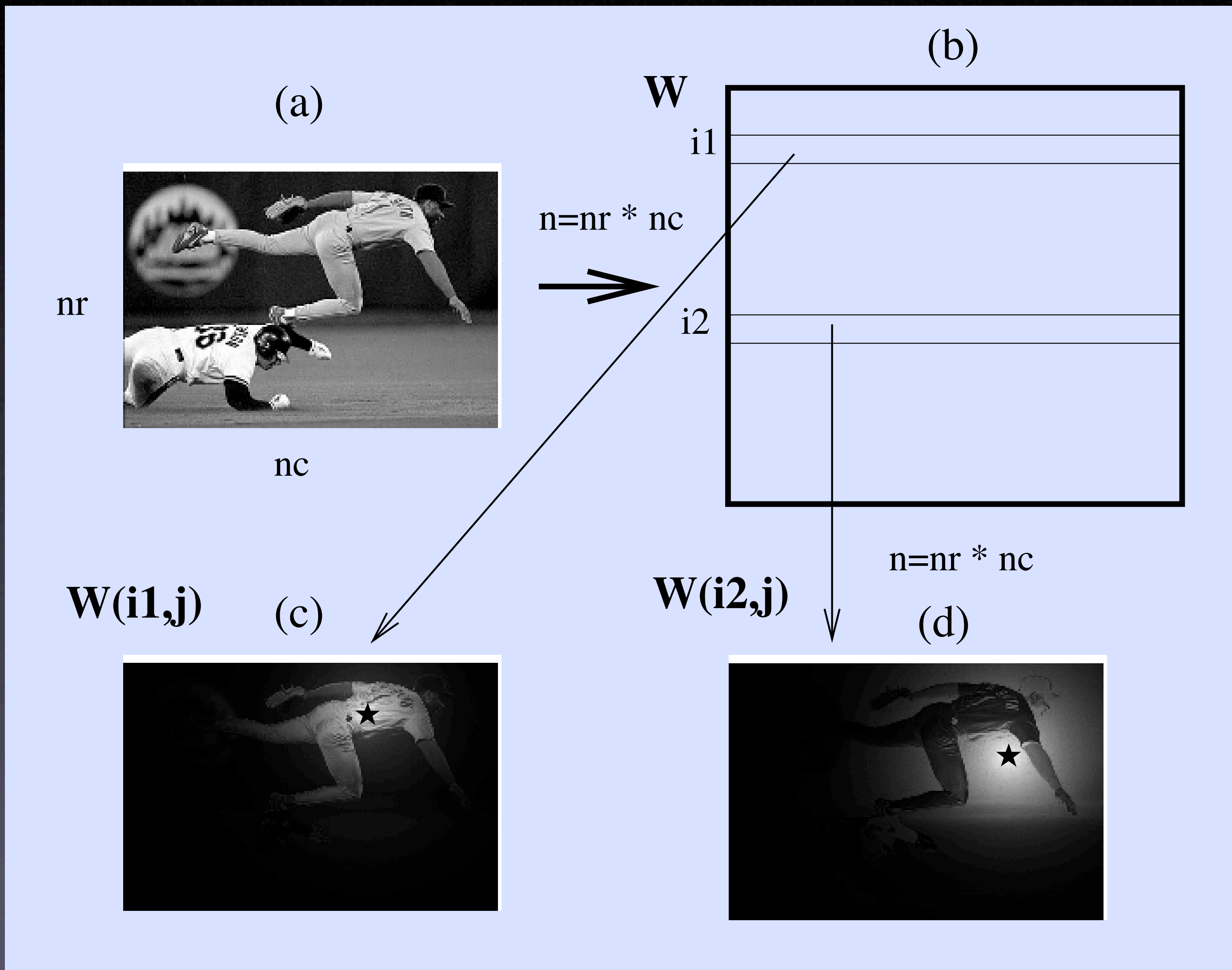


Pair-wise similarity matrix W

Laplacian matrix D-W

Degree matrix D: $D(i,i) = \sum_j W_{i,j}$

Graph weight matrix W

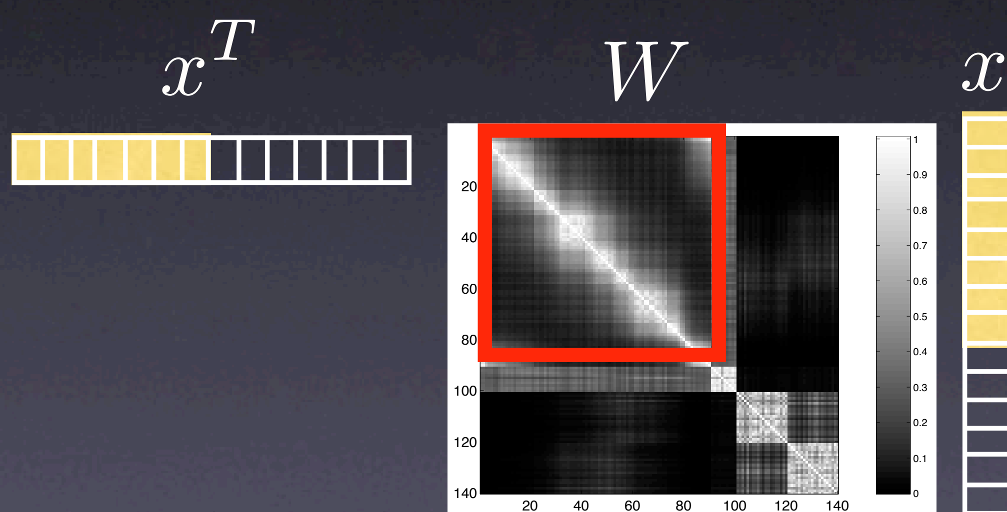


Laplacian matrix D-W

Let x = X(1,:) be the indicator of group 1

$$X = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

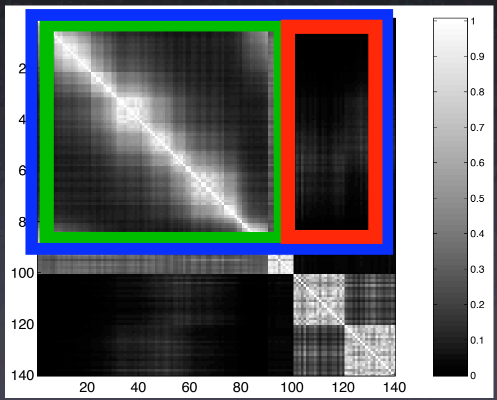
asso(A,A) = $x^T W x$



Laplacian matrix D-W

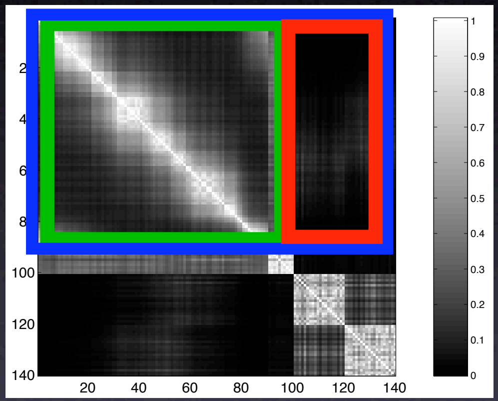
$x^T D x$ $\quad$ $x^T W x$

Cut(A,V-A) = vol(A) - asso(A,A)



$$Cut(A, V - A) = x^T (D - W) x$$

$$Ncut(X) = \frac{1}{K}\sum_{l=1}^{K}\frac{cut(V_l, V - V_l)}{vol(V_l)}$$



$$= \frac{1}{K}\sum_{l=1}^{K}\frac{X_l^T(D - W)X_l}{X_l^T D X_l}$$

$$X \in \{0,1\}^{N\times K}, X1_K = 1_N$$

Step 1: Find Continuous Global Optima

Scaled partition matrix. $Z = X(X^T DX)^{-\frac{1}{2}}$

$$X = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow Z = \begin{bmatrix} \frac{1}{\sqrt{vol(A)}} & 0 & 0 \\ \frac{1}{\sqrt{vol(A)}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{vol(B)}} & 0 \\ 0 & \frac{1}{\sqrt{vol(B)}} & 0 \\ 0 & 0 & \frac{1}{\sqrt{vol(C)}} \end{bmatrix}$$

Step I: Find Continuous Global Optima

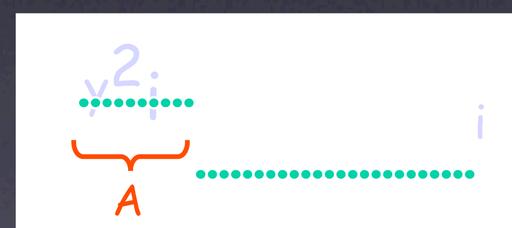
$$\text{Ncut} = \frac{1}{K}\sum_{l=1}^{K}\frac{X_l^T(D-W)X_l}{X_l^T D X_l}$$

becomes

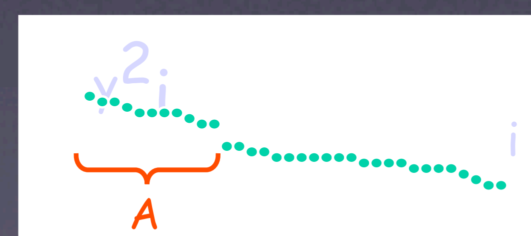
$$Ncut(Z) = \frac{1}{K}tr(Z^T W Z) \qquad Z^T D Z = I_K$$

Eigensolutions

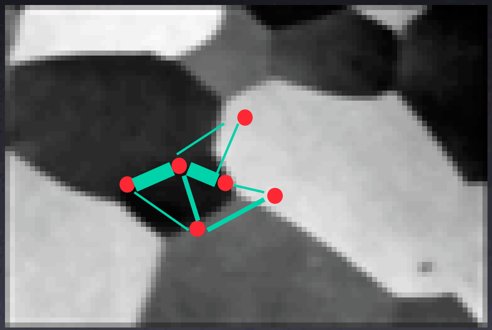
$$(D-W)z^* = \lambda D z^*$$

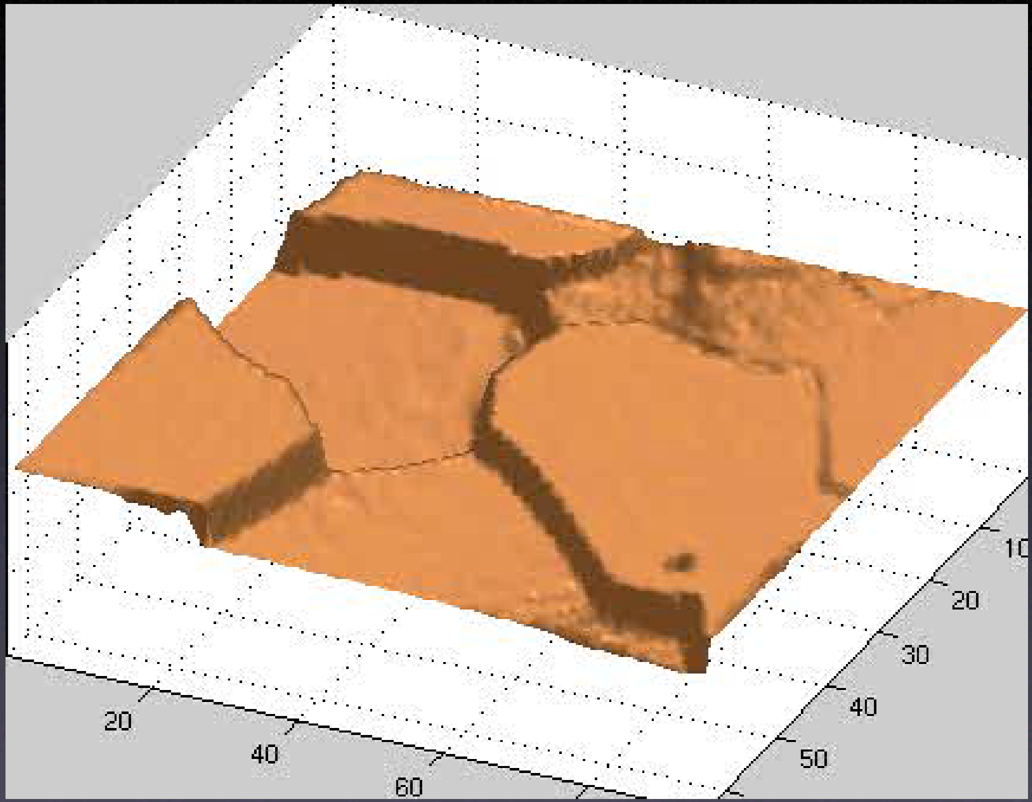


$$Z^* = [z_1^*, z_2^*, ..., z_k^*]$$

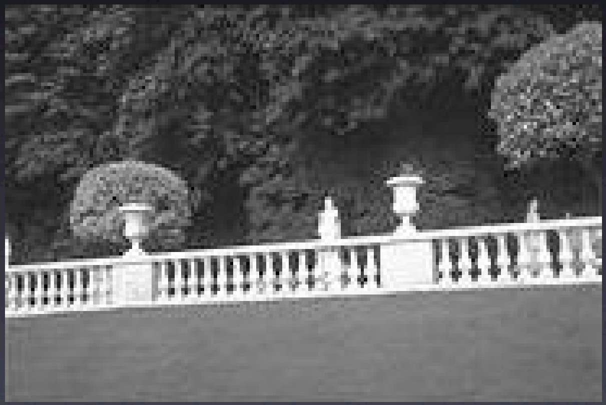


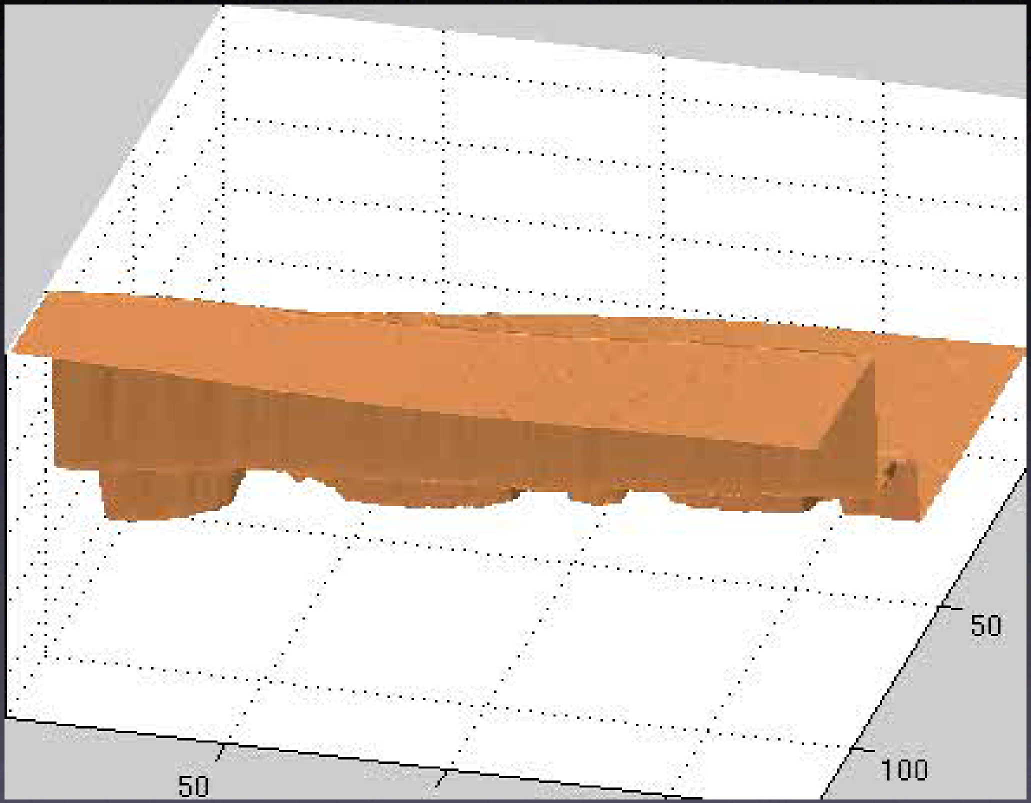
Interpretation as a Dynamical System





Interpretation as a Dynamical System





Step I: Find Continuous Global Optima

Partition

$$X = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

⇨ ⇦

Scaled Partition

$$Z = \begin{bmatrix} \frac{1}{\sqrt{vol(A)}} & 0 & 0 \\ \frac{1}{\sqrt{vol(A)}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{vol(B)}} & 0 \\ 0 & \frac{1}{\sqrt{vol(B)}} & 0 \\ 0 & 0 & \frac{1}{\sqrt{vol(C)}} \end{bmatrix}$$

⇨ ⇦

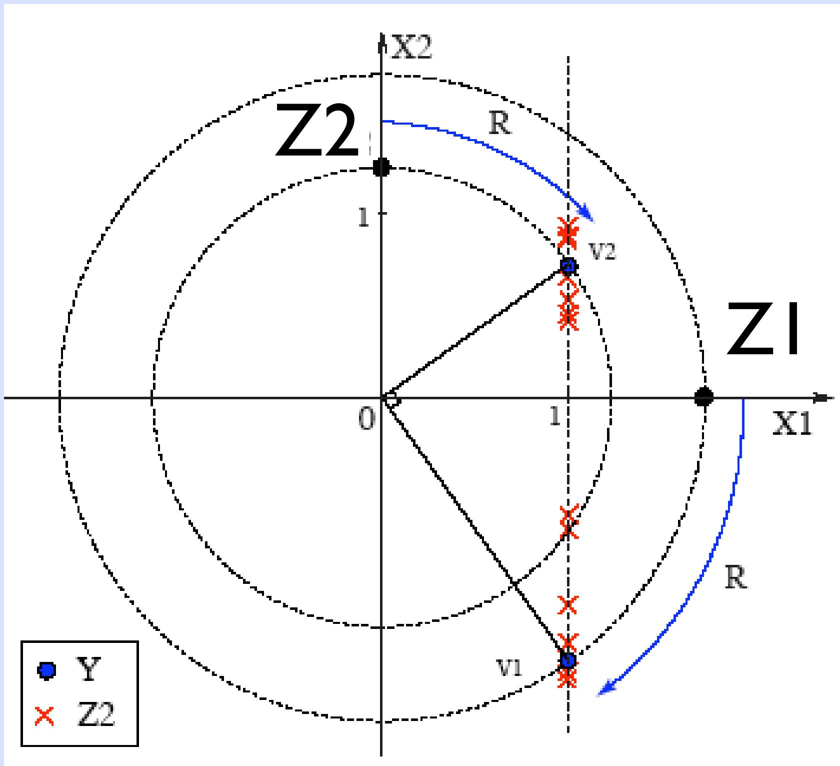
Eigenvector solution

$$Z^* = \begin{bmatrix} \frac{1}{\sqrt{vol(A)}} & 0 & 0 \\ \frac{1}{\sqrt{vol(A)}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{vol(B)}} & 0 \\ 0 & \frac{1}{\sqrt{vol(B)}} & 0 \\ 0 & 0 & \frac{1}{\sqrt{vol(C)}} \end{bmatrix} \times \begin{bmatrix} cos(\theta) & -sin(\theta) \\ sin(\theta) & cos(\theta) \end{bmatrix}$$

$$(D - W)Z^* = \lambda D Z^*$$

If Z* is an optimal, so is $\{ZR : R^T R = I_K\}$

Step II: Discretize Continuous Optima



Target partition

$$X = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}$$

Rotation R

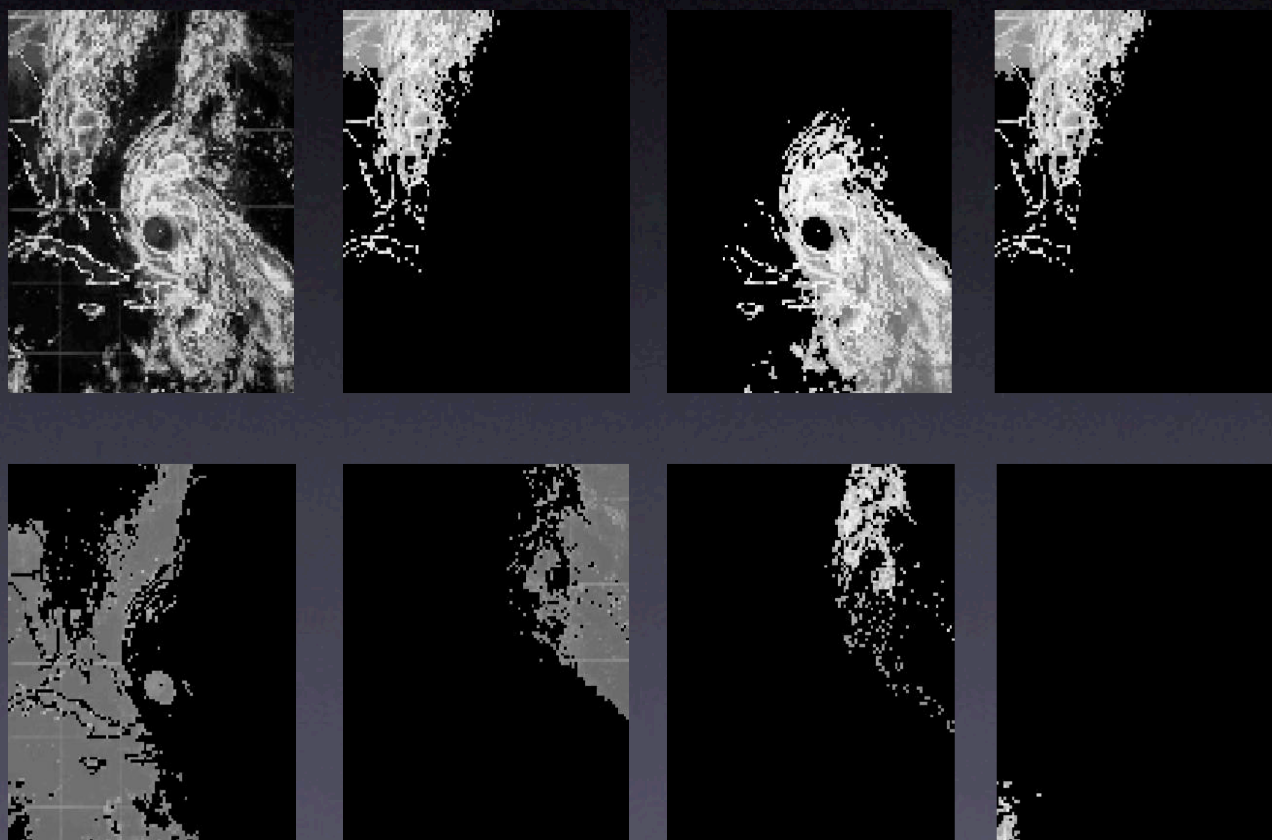
Eigenvector solution

$$Z^* = \begin{bmatrix} 1 & -1.4 \\ 1 & -1.3 \\ 1 & 0.8 \\ 1 & 0.9 \\ 1 & 0.7 \end{bmatrix}$$

Rotation R can be found exactly in 2-way partition

Brightness Image Segmentation

brightness image segmentation



Part II:
Segmentation Measurement, Benchmark