# MIDTERM EXAMINATION 

Networked Life (MKSE 112)
October 27, 2011
Prof. Michael Kearns
This is a closed-book exam. You should have no material on your desk other than the exam itself and a pencil or pen. If you run out of room on a page, you may use the back, but be sure to indicate you have done so.

Name: $\qquad$

Penn ID: $\qquad$

Problem 1: $\qquad$ /10

Problem 2: $\qquad$ /10

Problem 3: $\qquad$ /15

Problem 4: $\qquad$ /10

Problem 5: $\qquad$ /15

Problem 6: /15

Problem 7: $\qquad$ /15

Problem 8: $\qquad$ /10

TOTAL: $\qquad$ /100

Problem 1 ( 10 points) For each of the following statements, simply write "TRUE" or "FALSE".
a. The Preferential Attachment network formation model was introduced in order to explain high clustering coefficients in social networks.
b. The smallest number of edges a network with N vertices can have and still be connected is $\mathrm{N}-1$.
c. The paper "Graph Structure in the Web" cites the Strongly Connected Component of the Web as having approximately $91 \%$ of all pages.
d. In Kleinberg's model of navigation in networks, individuals may sometimes forward the message away from the target.
e. Standard notions of equilibrium do not require that individuals enjoy high satisfaction or payoffs.
f. The PageRank of a vertex is determined entirely by its own degree.
g. If you have K friends or neighbors in a social network, the number of possible friendships among your friends grows roughly like K.
h. No matter how big N is, it is always possible to create networks of diameter $\mathrm{N} / 2$ in which every vertex has small degree.
i. The distribution of distances travelled by dollar bills in "Where's George?" is approximately normal (bell-shaped).
j. Zipf's Law states that the k-th most frequent English word is about $1 / \mathrm{k}$ as frequent as the most frequent English word.


Problem 2 ( 10 points) For each of the two networks above, compute its clustering coefficient, and the equivalent edge density (value of p) in the Erdos-Renyi model. Say whether you think the network is highly clustered or not, and why.
(a) Network A
(b) Network B

Problem 3 ( 15 points) Describe a real-world, large-scale network (it can be social, technological, biological, economic, financial, etc.) that you believe does not have most or any of the main universal structural properties of typical social networks that we discussed in class and the readings. You should describe what the vertices and edges of your network are clearly, and then write a brief essay discussing why you think your network does not have the properties, and what might account for its difference with typical social networks.


Problem 4 ( 10 points) The image above is a screenshot from a simulator that we demonstrated in class. Briefly but clearly describe how this simulator works, including the underlying model it is illustrating. What is the main phenomenon about the underlying model that the simulator demonstrates? Be as detailed as possible. Then briefly explain in words why this phenomenon is common in large real-world social networks such as Facebook.

Problem 5 ( 15 points) Consider the following game of "competitive contagion" on networks. There are two competing players, Red and Blue. There is a network known to both players, and both players have some number of initial "seed" infections they can place in the network. Both players must choose where to place their seeds simultaneously. If both players choose to seed the same vertex, the vertex becomes infected with each color with probability $1 / 2$. Once the seeds are placed, uninfected vertices are updated according to the following rule: any vertex adjacent to a vertex already infected with Red or Blue becomes permanently infected with that same color. If an uninfected vertex has both Red and Blue neighbors, it becomes infected with Red or Blue with equal probability. Updates of uninfected vertices occur as soon as they have an infected neighbor.
(a) Suppose the network is a "hub and spokes" network, where vertex 1 is connected to vertices $2, \ldots, \mathrm{~N}$ and there are no other connections. Let each player have one seed infection. Describe the Nash equilibria of this game.
(b) Suppose the network is a simple cycle or ring of N vertices, and again let each player have one seed infection. Describe the Nash equilibria of this game.
(c) Suppose the network is again a cycle, and let the Red player have two seed infections and the Blue player one seed infection. Describe the Nash equilibria of this game.

## NWLife Forest Fire Demo



Problem 6 ( $\mathbf{1 5}$ points) The screenshot above shows the end result of running the forest fire simulator that we demonstrated in class.
(a) Briefly but clearly describe how this simulator works, including initialization and the process simulated.
(b) What value for the "Probability" (which has been blacked out) do you think was used for this particular run of the simulator? Why?
(c) Briefly but clearly describe how this simulator can be formalized as a model of random network formation, and the examination of a particular structural property (which you should name or describe) of the networks it generates.


Problem 7 ( 15 points) Consider the curve shown in the figure above, which is similar to diagrams we have analyzed in a recent lecture.
(a) Give an example of a real-world activity that might roughly correspond to the dynamics represented by this diagram.
(b) Consider a typical starting point to the left of the peak of the curve, such as the point labeled X. Describe in words how the attendance dynamics evolve from this starting point. You may want to annotate the diagram.
(c) Are there any equilibria in this system? If so, mark them on the diagram. Is it clear from the diagram whether or not the attendance will ever stabilize?

Problem 8 ( 10 points) For any value of N, describe a network of N vertices in which the diameter is as small as possible, the clustering coefficient is as high as possible, and there is at least one "connecter" vertex. The total number of edges in your network should grow proportionally (linearly) with N and not larger.
Describe your network as precisely and succinctly as possible, and give the values of the diameter and clustering coefficient. You are free to provide an illustrating diagram.

