MIDTERM EXAMINATION
Networked Life (NETS 112)
October 23, 2018
Prof. Michael Kearns
This is a closed-book exam. You should have no material on your desk other than the exam itself and a pencil or pen. If you run out of room on a page, you may use the back, but be sure to indicate you have done so. You may also make annotations directly on any diagrams given.

Name:

Problem 1: $\qquad$ /10

Problem 2: $\qquad$ /10

Problem 3: $\qquad$ /15

Problem 4: $\qquad$ /15

Problem 5: $\qquad$ /15

Problem 6: $\qquad$ /10

Problem 7: $\qquad$ /10

Problem 8: /15
$\qquad$ /100

Problem 1 (10 points). Next to each lettered item on the left, write the number of the item on the right that is the best match. It might help to do the ones that are most obvious to you first.
(a) Facebook recruiting

1. Travers and Milgram
(b) few neurons
2. tennis ball
(c) cascade tree
3. C. Elegans
(d) Sharon, MA
(e) no cycles
(f) Erdos-Renyi
4. virality calculations
(g) audience participation
(h) overfitting
5. structural diversity
(i) highest virality
6. heavy tails
(j) file sizes
7. Preferential Attachment

Problem 2 (10 points). Consider the network in which there is a vertex for each word in the English language, and there is an edge between two words if they have appeared together in the same sentence in a New York Times article published in the last year. For example, there would be an edge between the words "dissident" and "journalist" due to a sentence in Saturday's NYT, but it might be quite unlikely there is an edge between (say) "alpine" and "dissident".

For each of the universal network properties discussed in class and readings, indicate whether you think this network will exhibit that property, and briefly but clearly justify your answer.

Problem 3 ( 15 points). Consider the network shown below.

(a) Ignoring the dotted line, calculate the clustering coefficient of this network.
(b) Calculate the overall edge density of this network.
(c) Calculate the edge density on just the left side of the dotted line, and also on just the right side of the dotted line. Exclude the edges crossing the dotted line, as they are neither on the left side nor the right side.
(d) Calculate the edge density between the vertices on the left and right sides of the dotted line.
(e) Based on your calculations above and class discussions, would you say this network is highly clustered or not? Briefly but precisely justify your answer.

Problem 4 ( 15 points). The table below is reproduced from Wikipedia, and was discussed in class. In it, $\boldsymbol{k}$ refers to (worst-case) diameter, and $\boldsymbol{d}$ refers to degree.

| $\boldsymbol{d}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ | $\mathbf{9}$ | $\mathbf{1 0}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{3}$ | 10 |  |  | 70 | 132 | 196 | 336 | 600 | 1250 |
| $\mathbf{4}$ |  | 41 | 98 | 364 | 740 | 1320 | 3243 | 7575 | 17703 |
| $\mathbf{5}$ |  | 72 | 212 | 624 | 2772 | 5516 | 17030 | 57840 | 187056 |
| $\mathbf{6}$ | 32 | 111 | 390 | 1404 | 7917 | 19383 | 76461 | 331387 | 1253615 |
| $\mathbf{7}$ | 50 | 168 | 672 | 2756 | 11988 | 52768 | 249660 | 1223050 | 6007230 |
| $\mathbf{8}$ | 57 | 253 | 1100 | 5060 | 39672 | 131137 | 734820 | 4243100 | 24897161 |
| $\mathbf{9}$ | 74 | 585 | 1550 | 8268 | 75893 | 279616 | 1697688 | 12123288 | 65866350 |
| $\mathbf{1 0}$ | 91 | 650 | 2286 | 13140 | 134690 | 583083 | 4293452 | 27997191 | 201038922 |
| $\mathbf{1 1}$ | 104 | 715 | 3200 | 19500 | 156864 | 1001268 | 7442328 | 72933102 | 600380000 |
| $\mathbf{1 2}$ | 133 | 786 | 4680 | 29470 | 359772 | 1999500 | 15924326 | 158158875 | 1506252500 |
| $\mathbf{1 3}$ | 162 | 851 | 6560 | 40260 | 531440 | 3322080 | 29927790 | 249155760 | 3077200700 |
| $\mathbf{1 4}$ | 183 | 916 | 8200 | 57837 | 816294 | 6200460 | 55913932 | 600123780 | 7041746081 |
| $\mathbf{1 5}$ | 187 | 1215 | 11712 | 76518 | 1417248 | 8599986 | 90001236 | 1171998164 | 10012349898 |
| $\mathbf{1 6}$ | 200 | 1600 | 14640 | 132496 | 1771560 | 14882658 | 140559416 | 2025125476 | 12951451931 |

a) As clearly and precisely as you can, explain what the numbers in the table are measuring. Do the numbers in the table represent empirical measurements, or are they from mathematical theories?
b) What can you observe about the growth rate of the numbers? Explain why it makes broad sense.
c) Consider the entry for 7041746081 , which is approximately the size of the world population. Are the values for which this entry is achieved consistent with the readings from class? Justify your answer.

Problem 5 ( 15 points). Consider the network shown below.

(a) What is its worst-case diameter?
(b) Clearly add two edges so that the worst-case diameter decreases by exactly 1.
(c) Draw a network of 5 vertices and 6 edges such that the worst-case diameter is 3 , and when you remove 2 edges of your choice (clearly marked on your drawing), the worst-case diameter becomes 4 .

Problem 6 ( 10 points). Fill in the following table by writing either "yes" or "no" in each entry.

|  | Erdos-Renyi | Preferential Attachment |
| :--- | :--- | :--- |
| Giant Component |  |  |
| Small Diameter |  |  |
| Sparsity |  |  |
| Heavy Tails |  |  |
| High Clustering |  |  |

## Problem 7 (10 points).



| Time | Nodes who shared |
| :--- | :--- |
| 1 | A |
| 2 | B, C |
| 3 | D |
| 4 | H |
| 5 | E |
| 6 | F |
| 7 | G |

(a) Draw the cascade tree for network above and the given timestamps of when each vertex shared. Break ties by attributing the cause of resharing to the neighbor who shared most recently.
(b) Do the same, except break ties by attributing the cause of resharing to the neighbor who shared it first.


| Time | Nodes who shared |
| :--- | :--- |
| 1 | C |
| 2 | E |
| 3 | D |
| 4 | F |
| 5 | A |
| 6 | G |

(c) Draw the cascade tree for network above and the given timestamps of when each vertex shared. Break ties by attributing the cause of resharing to the neighbor who shared most recently.
(d) Compute the virality of the cascade tree for part (c).

Problem 8 ( $\mathbf{1 5}$ points). One of the assigned readings begins with the following passage:
"A crucial task in the analysis of on-line social-networking systems is to identify important people — those linked by strong social ties — within an individual's network neighborhood. Here we investigate this question for a particular category of strong ties, those involving spouses or romantic partners."
(a) Briefly but clearly describe what the source of data for the study was. Be sure to indicate exactly what information was used, and what information was not used.
(b) Briefly but clearly describe the question or hypothesis the authors were investigating.
(c) Briefly but clearly describe the findings or results of the study.

