Homework 2
Networked Life (NETS 112)
Fall 2019
Prof Michael Kearns

Posted November 29, 2019. In order to give people flexibility it will be due in hard-copy format on Thursday, December 19 (which is the last day of final exams), but you can turn it in anytime before, including at our final exam on December 12. Details about where to turn in the homework will be announced.

Collaboration of any kind is NOT permitted on the homework.

Your Name:

Problem 1. Give a concrete (numerical) example of a 2-player, 3-action game in which there is no pure Nash equilibrium, and in which (1/3, 1/3, 1/3) is *not* a mixed Nash equilibrium (and thus your game is not equivalent to Rock-Paper-Scissors). Give the (numerical) mixed Nash equilibrium of your game, and argue why there is no pure strategy equilibrium.

Problem 2. Give an example of a connected network with 10 vertices in which the minimum number of colors required for a proper coloring is exactly 3. (Recall that a proper coloring is an assignment of colors to vertices such that every vertex is a different color than all of its neighbors.) Show a proper coloring using 3 colors. Then show a Nash equilibrium of the coloring game on your network that is not a proper coloring.

Problem 3. Consider diagrams of the kind discussed in lecture and in Schelling's "Micromotives and Macrobehavior" for modeling activities in which the number of people participating at the current round (the y axis) is a function of the number participating at the last round (the x axis).

a. Carefully draw such a diagram in which there are 3 stable equilibria and 2 unstable equilibria.

b. Carefully draw such a diagram in which there are 3 stable equilibria and 4 unstable equilibria.

c. What can you say in general about the relationship between the number of unstable and stable equilibria in such models?

Problem 4. Consider networks in which there are two types of individuals/vertices. Red vertices are happy if at least half of their neighbors are also Red, and otherwise are unhappy. Blue vertices are happy if at least half of their neighbors are also Blue, and otherwise are unhappy.

a. Draw a connected network in which there are 5 Red vertices and 5 Blue vertices, and the total happiness is as large as possible.

b. Draw a connected network in which there are 5 Red vertices and 5 Blue vertices, and the total happiness is as small as possible.

c. Draw a connected network in which there are 5 Red vertices and 5 Blue vertices, and only the Red vertices are happy.

d. Draw a connected network in which there are 5 Red vertices and 5 Blue vertices, and there are exactly 3 happy Red vertices, and exactly 3 happy Blue vertices.

Problem 5. Write a 2-3 page single-spaced essay in which you summarize the main themes and arguments in Chapter 4 ("Lost in the Garden") of "The Ethical Algorithm". Be sure to discuss the problems identified generally in the chapter, how they play out in scientific research, and some of the potential solutions discussed.