

# **Networked Games: Coloring, Consensus and Voting**

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**Networked Life**

**NETS 112**

**Fall 2019**

# Experimental Agenda

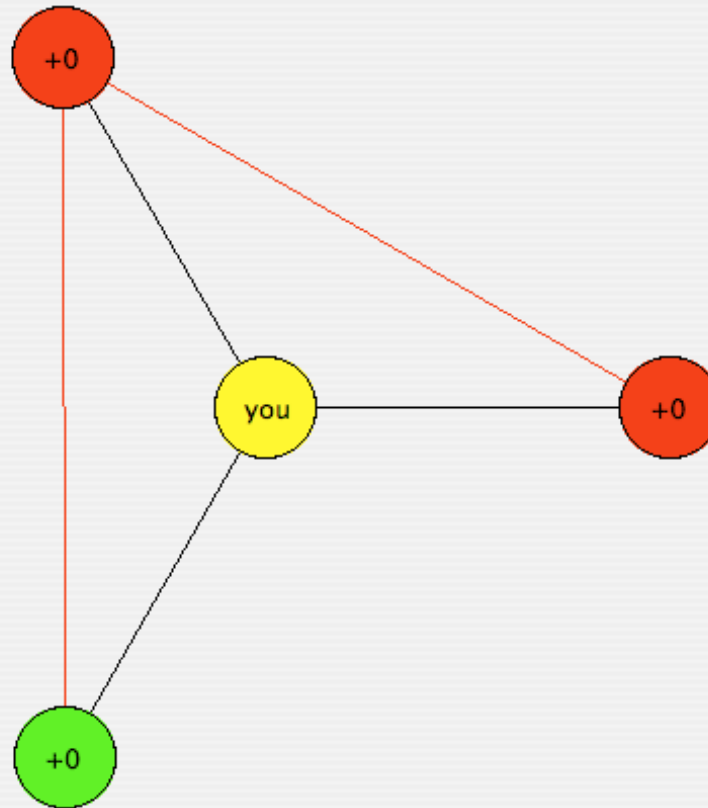
- Human-subject experiments at the intersection of CS, economics, sociology, “network science”
- Subjects simultaneously participate in groups of ~ 36 people
- Subjects sit at networked workstations
- Each subject controls some simple property of a single vertex in some underlying network
- Subjects have only *local* views of the activity: state of their own and neighboring vertices
- Subjects have (real) financial incentive to solve their “piece” of a collective (global) task
- Simple example: graph coloring (social differentiation)
  - choose a color for your vertex from fixed set
  - paid iff your color differs from all neighbors when time expires
  - max welfare solutions = proper colorings
- Across many experiments, have deliberately varied *network structure* and *task/game*
  - networks: inspired by models from network science (small worlds, preferential attachment, etc.)
  - tasks: chosen for diversity (cooperative vs. competitive) and (centralized) computational difficulty
- Goals:
  - structure/tasks → performance/behavior
  - individual & collective modeling → prediction
  - computational and equilibrium theories

# Experiments to Date

- **Graph Coloring**
  - **player controls:** color of vertex; number of choices = chromatic number **payoffs:** \$2 if different color from all neighbors, else 0 **max welfare states:** optimal colorings **centralized computation:** hard even if approximations are allowed
- **Consensus**
  - **player controls:** color of vertex from 9 choices **payoffs:** \$2 if same color as all neighbors, else 0 **max welfare states:** global consensus of color **centralized computation:** trivial
- **Independent Set**
  - **player controls:** decision to be a “King” or a “Pawn”; variant with King side payments allowed **payoffs:** \$1/minute for Solo King; \$0.50/minute for Pawn; 0 for Conflicted King; continuous accumulation **max welfare states:** maximum independent sets **centralized computation:** hard even if approximations are allowed
- **Exchange Economy**
  - **player controls:** limit orders offering to exchange goods **payoffs:** proportional to the amount of the other good obtained **max welfare states:** market clearing equilibrium **centralized computation:** at the limit of tractability (LP used as a subroutine)
- **Biased Voting**
  - **player controls:** choice of one of two colors **payoffs:** only under global agreement; different players prefer different colors **max welfare states:** all red and all blue **centralized computation:** trivial
- **Networked Bargaining**
  - **player controls:** offers on each edge to split a cash amount; may have hidden deal limits and “transaction costs” **payoffs:** on each edge, a bargaining game --- payoffs only if agreement **max welfare states:** all deals/edges closed **centralized computation:** nontrivial, possibly difficult
- **Voting with Network Formation**
  - **player controls:** edge purchases and choice of one of two colors **payoffs:** only under global agreement; different players prefer different colors **max welfare states:** ??? **centralized computation:** ???

# Coloring and Consensus

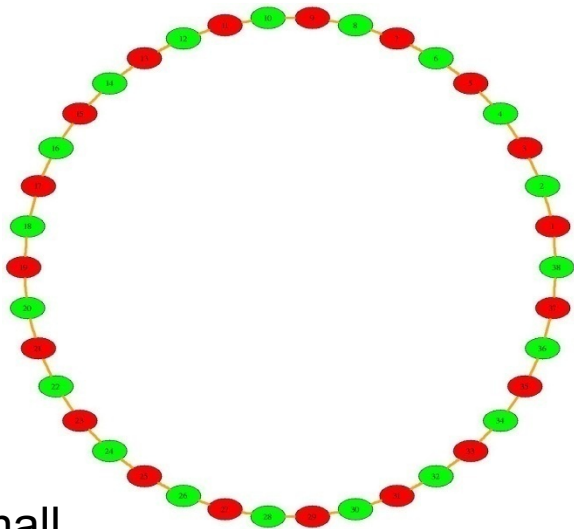
game progress: 80%  
game status: ColoringGame in progress  
elapsed time:  
your current payoff: **\$2.00**  
(payoff is \$2.00 if your color is DIFFERENT from all your neighbours, otherwise \$0.00)



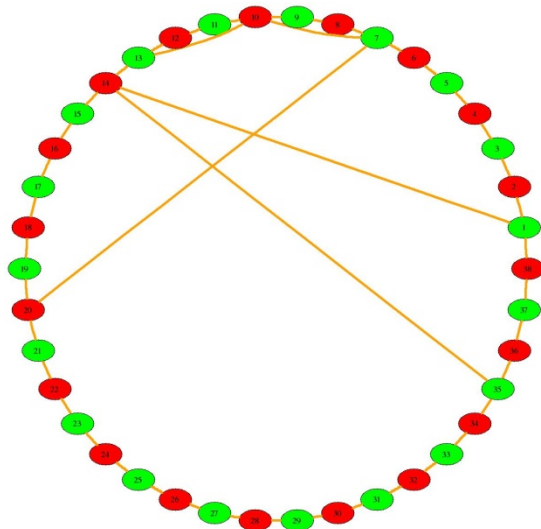
“first neighborhood” view

your color:  yellow  red  green

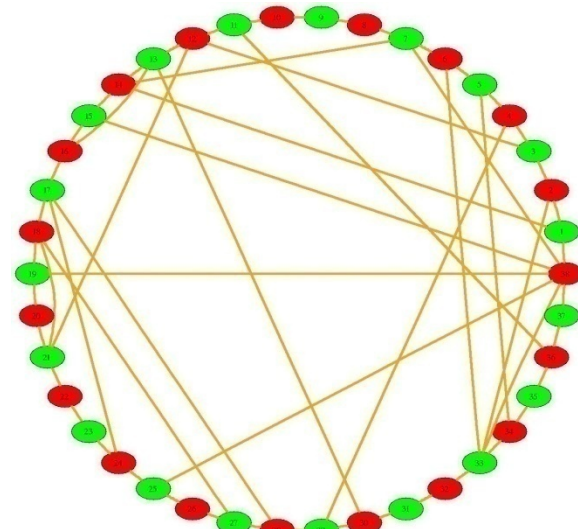
[\[demo\]](#)



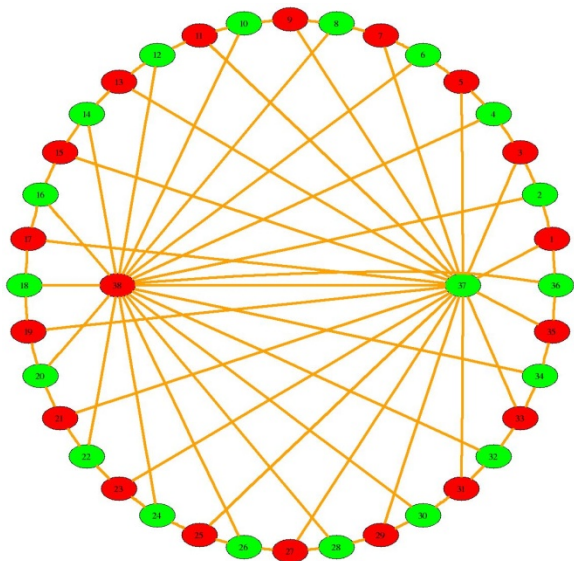
Simple Cycle



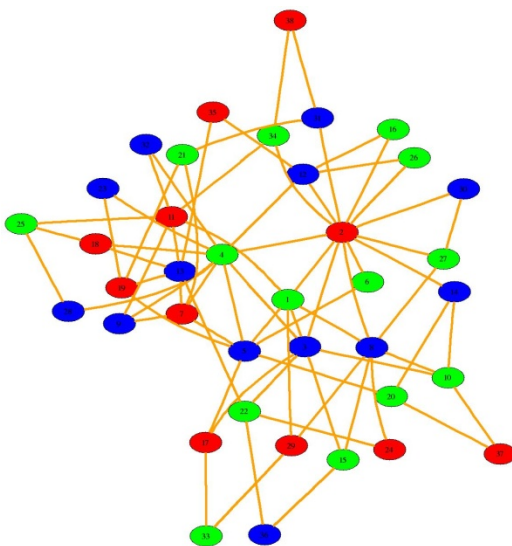
5-Chord Cycle



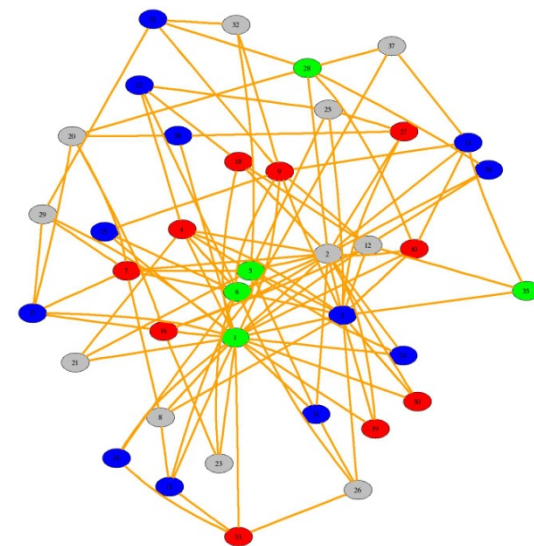
20-Chord Cycle



Leader Cycle



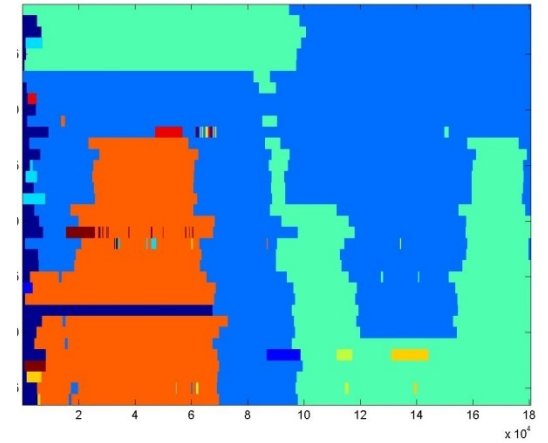
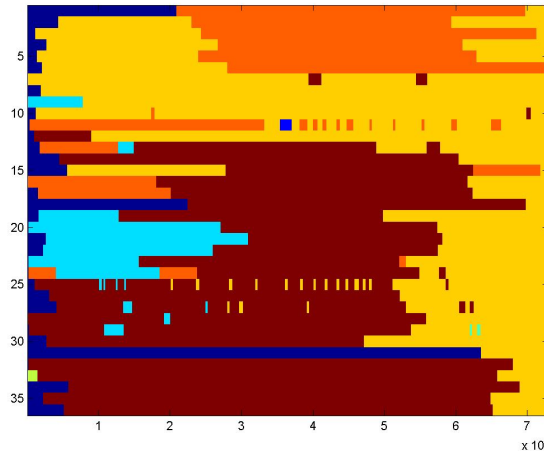
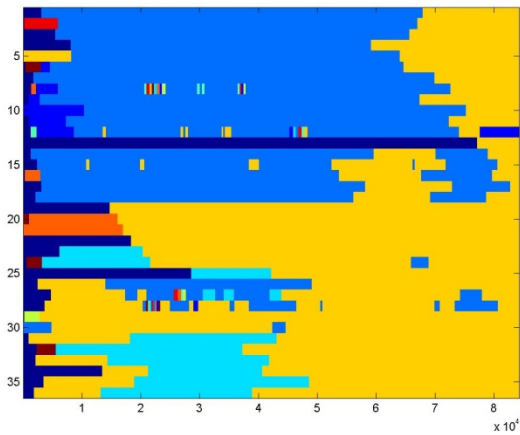
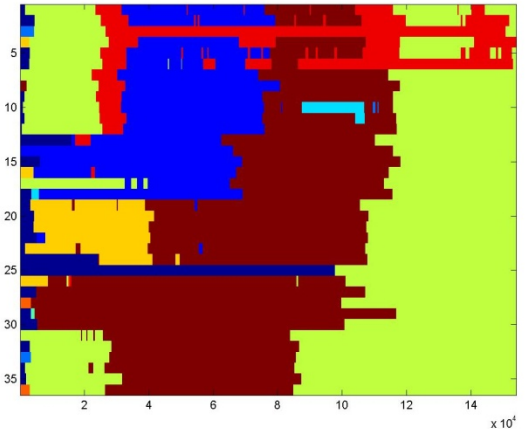
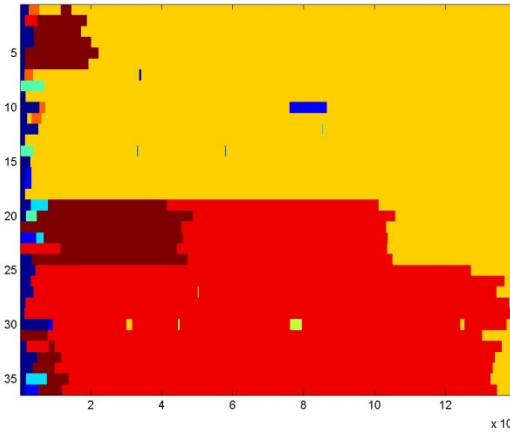
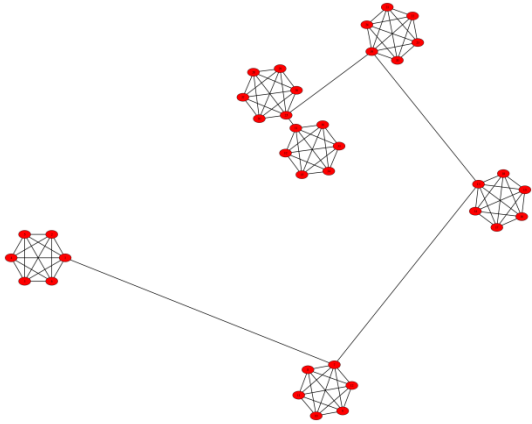
Preferential Attachment,  
 $\nu = 2$



Preferential Attachment,  
 $\nu = 3$

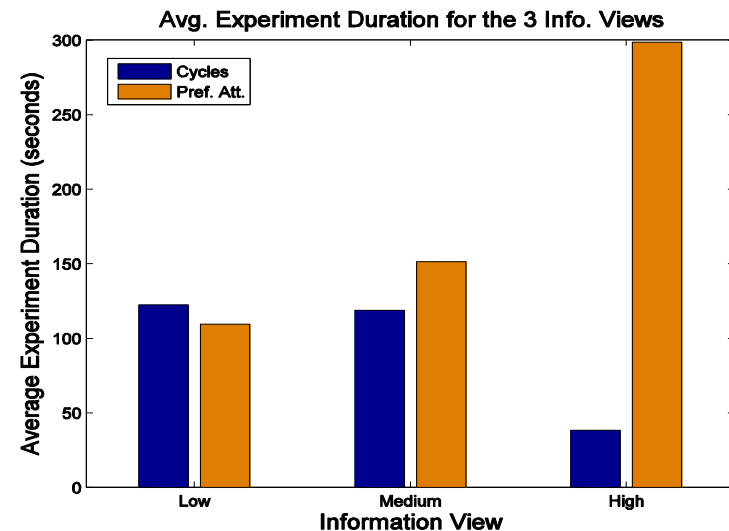
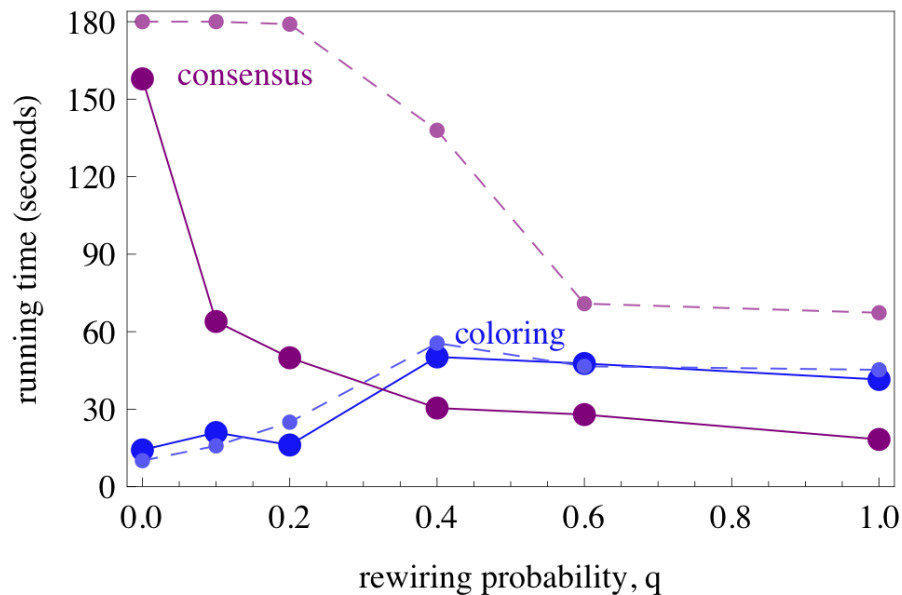
Small  
Worlds  
Family

# Art by Consensus



# Sample Findings

- Generally strong collective performance
  - nearly all problems globally solved in a couple minutes or less
- Systematic effects of structure on performance and behavior:
  - *rewiring harms* coloring performance in “clique chain” family
  - *rewiring helps* consensus performance in clique chain family
- Preferential attachment much harder than small worlds for coloring
  - natural heuristics can give reverse order of difficulty
- Providing more global views of activity:
  - helps coloring performance in small world family
  - harms coloring performance in preferential attachment
- Coloring problems solved more rapidly than consensus
  - easier to get people to disagree than agree





# Biased Voting in Networks

# Biased Voting in Networks

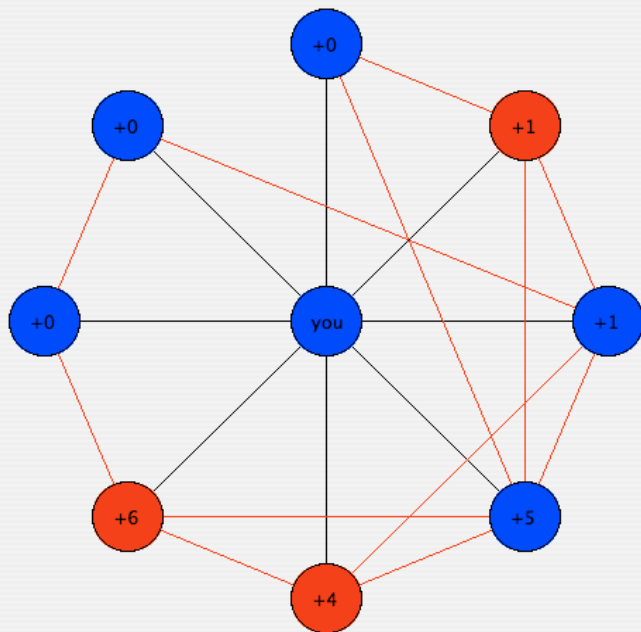
- Cosmetically similar to consensus, with a crucial strategic difference
- Deliberately introduce a tension between:
  - individual preferences
  - desire for collective unity
- Only two color choices; challenge comes from competing incentives
- If everyone converges to same color, everyone gets some payoff
- But different players have different preferences
  - each player has payoffs for their preferred and non-preferred color
  - e.g. \$1.50 red/\$0.50 blue vs. \$0.50 red/\$1.50 blue
  - can have symmetric and asymmetric payoffs
- High-level experimental design:
  - choice of network structures
  - arrangement of types (red/blue prefs) & strengths of incentives
  - most interesting to coordinate network structure and types

### Democratic Primary Games

Zak Xavier

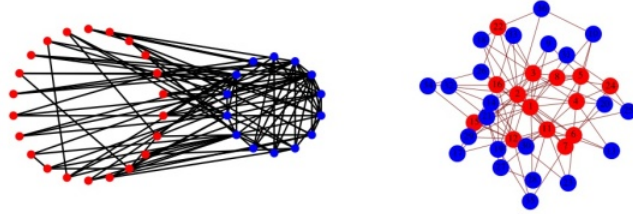
game progress:   
game status:   
elapsed time:

If unanimity is reached, your payoff will be  
**\$0.75 for red, \$1.25 for blue**

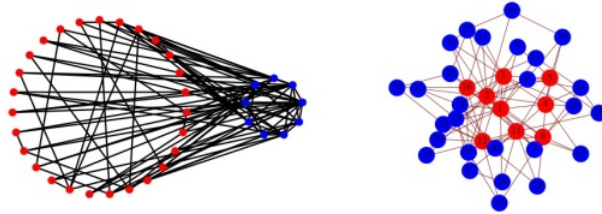


your color:  blue  red

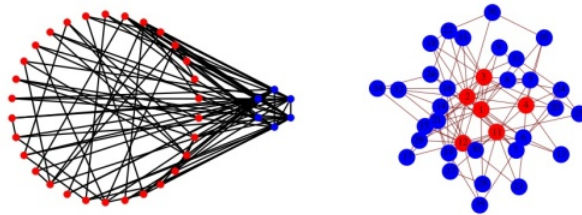
power22



power27



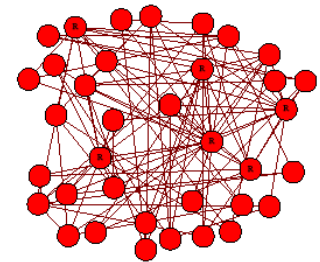
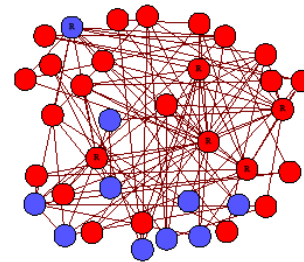
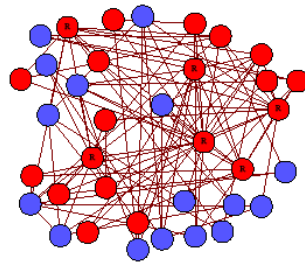
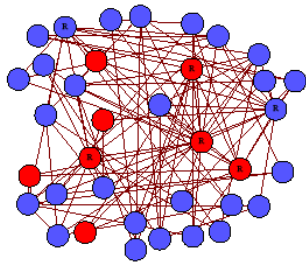
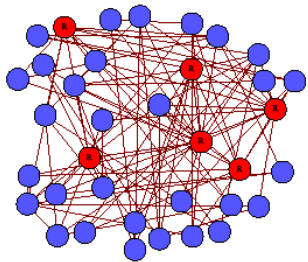
power30



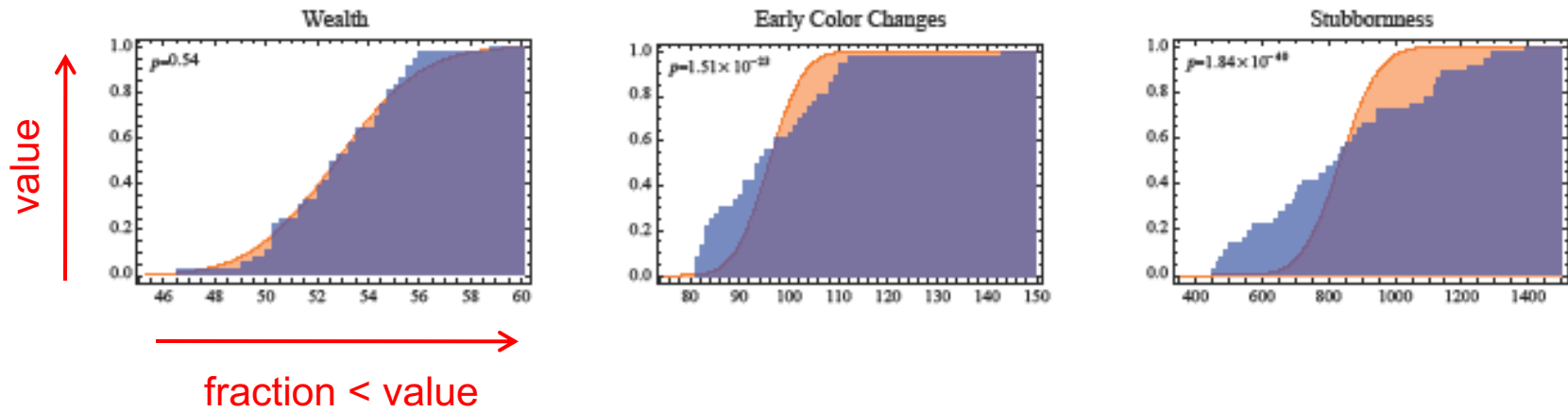
Minority Power: Preferential Attachment

# Summary of Findings

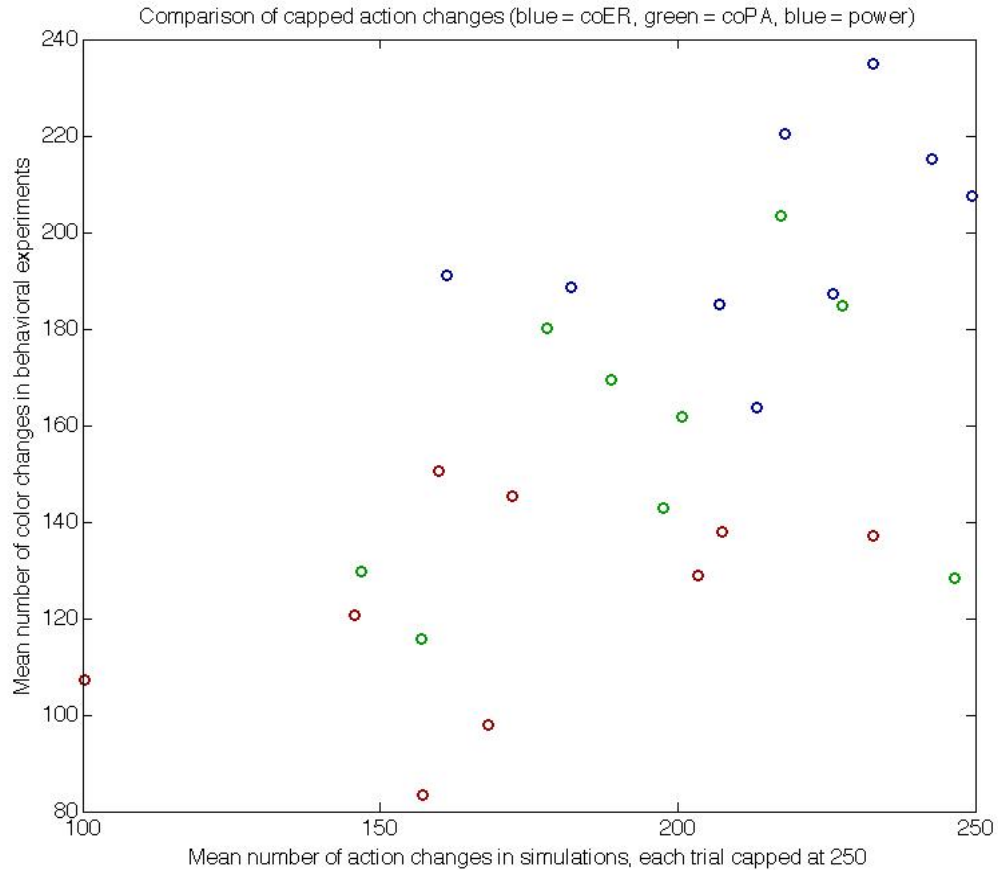
- 55/81 experiments reached global consensus in 1 minute allowed
  - mean of successful ~ 44s
- Effects of network structure:
  - Cohesion harder than Minority Power: 31/54 Cohesion, 24/27 Minority Power
  - all 24 successful Minority Powers converge to minority preference!
  - Cohesion P.A. (20/27) easier than Cohesion E-R
  - overall, P.A. easier than E-R (contrast w/coloring)
  - within Cohesion, increased inter-group communication helps
    - some notable exceptions...
- Effects of incentives:
  - asymmetric beats weak symmetric beats strong symmetric
  - the value of “extremists”



# Effects of “Personality”



# Behavioral Modeling



model: play color  $c$  with probability  $\sim \text{payoff}(c) \times \text{fraction in neighborhood playing } c$

# Lessons Learned, 2005-2011

- At least for  $n=36$ , human subjects remarkably good
  - diverse set of collective tasks
  - diverse set of network topologies
  - efficiency  $\sim 90\%$  across all tasks/topologies
- Network structure matters; interaction with **task**
  - contrast with emphasis on topology alone
- Importance of subject variability and style/personality
- Most recently: **endogenized** creation of the network
  - network formation games
  - challenging computationally (best response) and analytically



## Behavioural Network Games

Baffle Blythe

game status:

Game in progress

elapsed time:

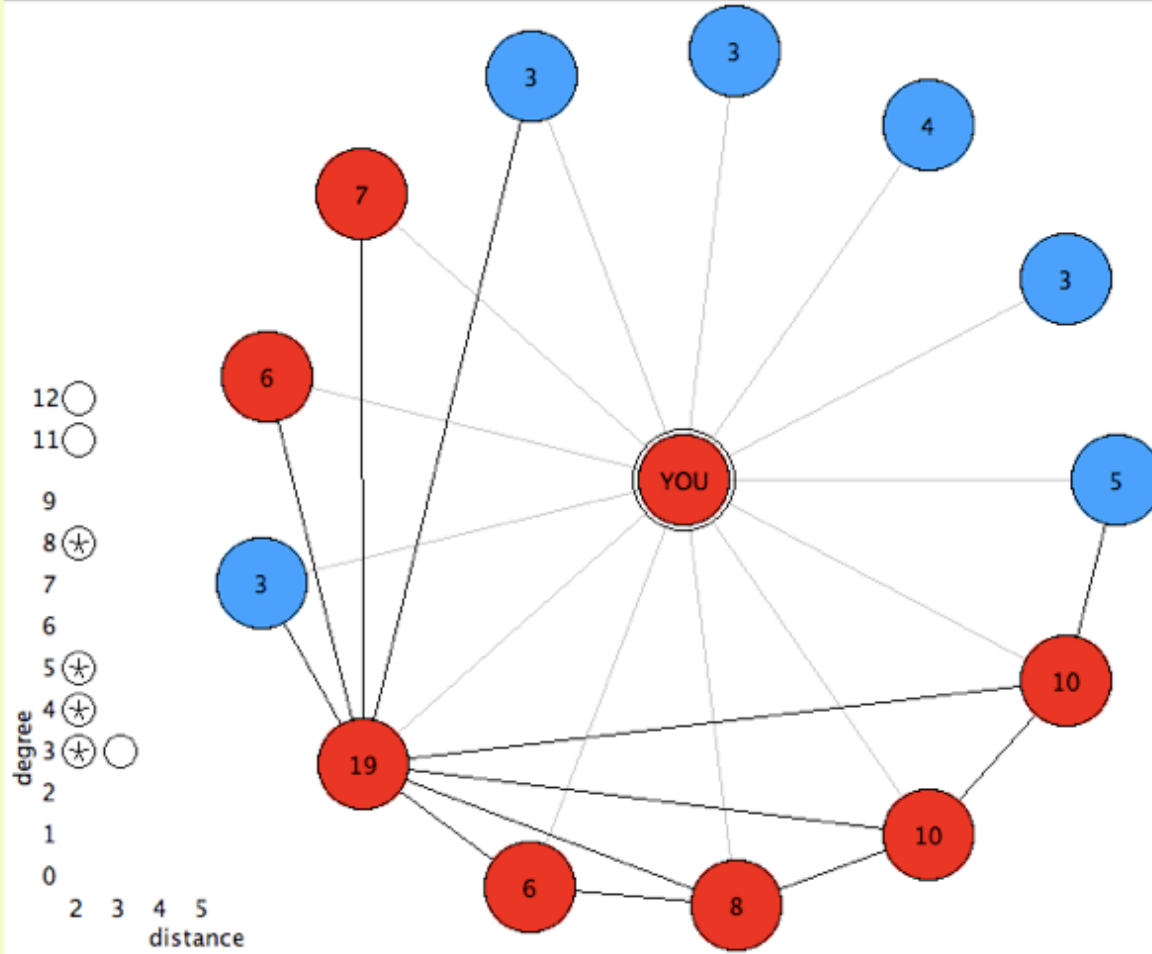
payoffs for unanimity:

\$3.00

\$2.00

edges cost \$0.03

You may purchase 35 edges.



your color:

red

blue

# Edge Purchases: Strategic Tensions

- Buy edges or not?
- For information or influence?
- Early in the game or late?
- To high degree or low degree players?
- Nearby or far away?

# Experimental Design

- Session A: 99 experiments
  - 63 “unseeded” with varying payoffs, imbalances, asymmetries
  - 36 seeded with Minority Power settings
- Session B: 72 experiments
  - mixture of unseeded and variety of seeded (cliques, torus)
- A: 47/99 solved (47%): 25/63 unseeded, MP 22/36
- B: 27/72 solved (38%)
- Session C: 72 experiments
  - **final networks** from “hard” settings in Session A
  - permitted 0 or 1 edge purchases per player
  - started with both **initial and final incentives** from Session A
- C: 25/72 (35%); All: 99/243 (41%)
- Subjects seem to build difficult networks!