Chapter 3
Digital Logic Structures

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Transistor: Building Block of Computers

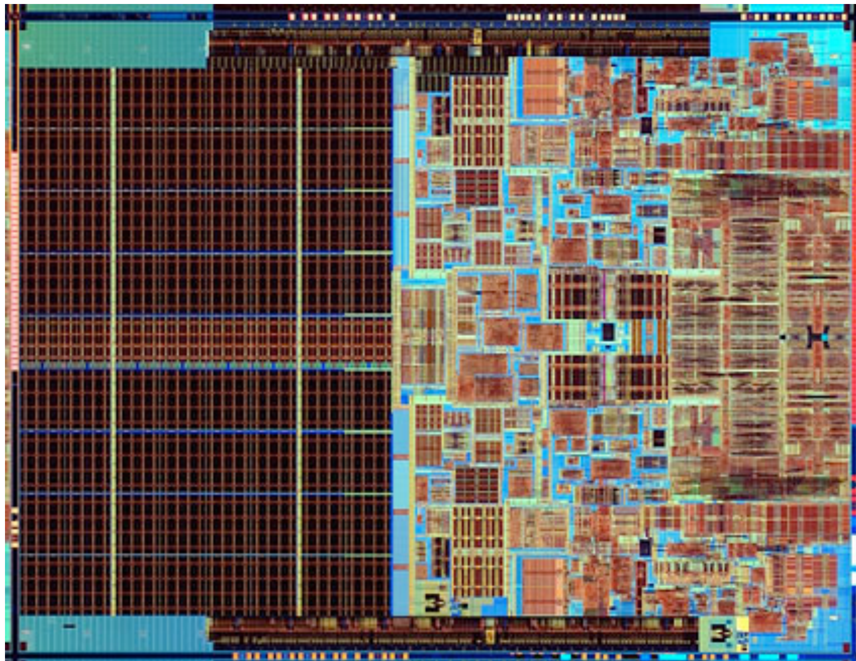
Microprocessors contain millions of transistors
- Intel Pentium 4 (2000): 48 million
- IBM PowerPC 750FX (2002): 38 million
- IBM PowerPC G5 (2003): 58 million
- **Intel Core Duo 2 (2006): 291 million** (192+ million in cache alone)

Logically, each transistor acts as a switch

Combined to implement logic functions
- **AND, OR, NOT**

Combined to build higher-level structures
- **Adder, multiplexer, decoder, register, …**

Combined to build processor
- **LC-3**



How do we represent data in a computer?

At the lowest level, a computer has electronic "plumbing"
- **Operates by controlling the flow of electrons**

Easy to recognize two conditions:
1. **Presence of a voltage – we'll call this state "1"**
2. **Absence of a voltage – we'll call this state "0"**

Computer use transistors as switches to manipulate bits
- **Before transistors: tubes, electro-mechanical relays (pre 1950s)**
- **Mechanical adders (punch cards, gears) as far back as mid-1600s**

Before describing transistors, we present an analogy…

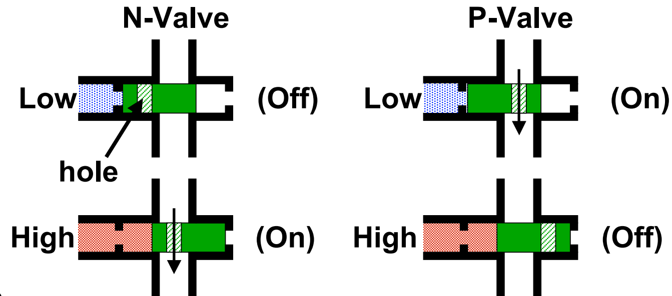
A Transistor Analogy: Computing with Air

Use air pressure to encode values

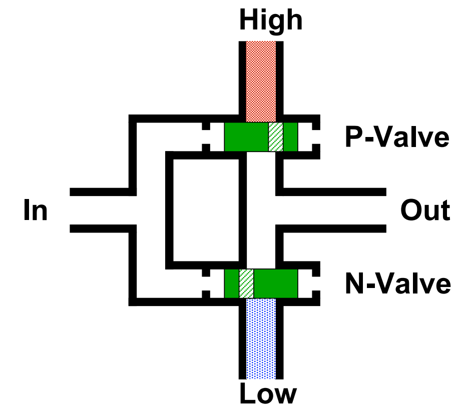
- High pressure represents a "1" (blow)
- Low pressure represents a "0" (suck)

Valve can allow or disallow the flow of air

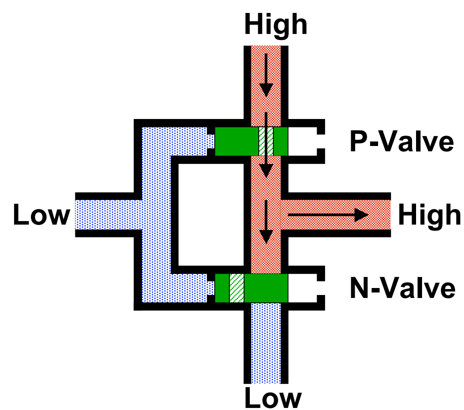
- Two types of valves



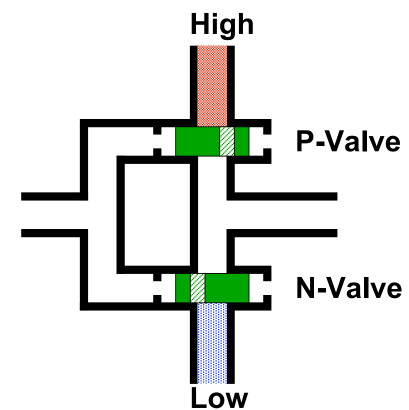
Pressure Inverter



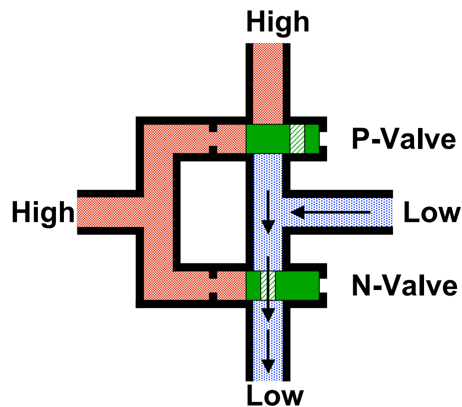
Pressure Inverter (Low to High)



Pressure Inverter



Pressure Inverter (High to Low)



Analogy Explained

Pressure differential → electrical potential (voltage)

- Air molecules → electrons
- High pressure → high voltage
- Low pressure → low voltage

Air flow → electrical current

- Pipes → wires
- Air only flows from high to low pressure
- Electrons only flow from high to low voltage
- Flow only occurs when changing from 1 to 0 or 0 to 1

Valve → transistor

- The transistor: one of the century's most important inventions

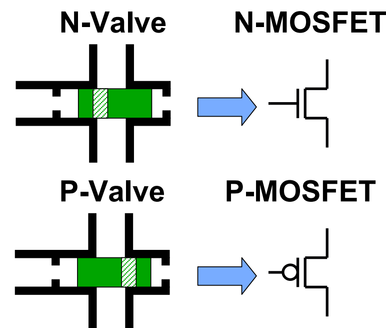
Transistors as Switches

Two types

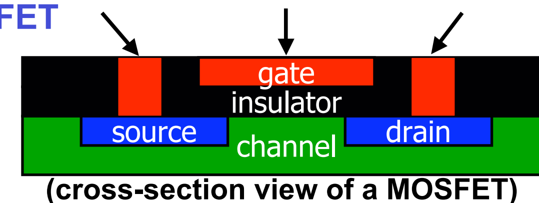
- N-type
- P-type

Properties

- Solid state (no moving parts)
- Reliable (low failure rate)
- Small (90nm channel length)
- Fast (<0.1ns switch latency)



MOS + FET



(cross-section view of a MOSFET)

MOS: three materials needed to make a transistor

- Metal (Al, W, Cu): conductor
- Oxide (SiO_2): insulator
- Semiconductor (doped Si): conducts under certain conditions

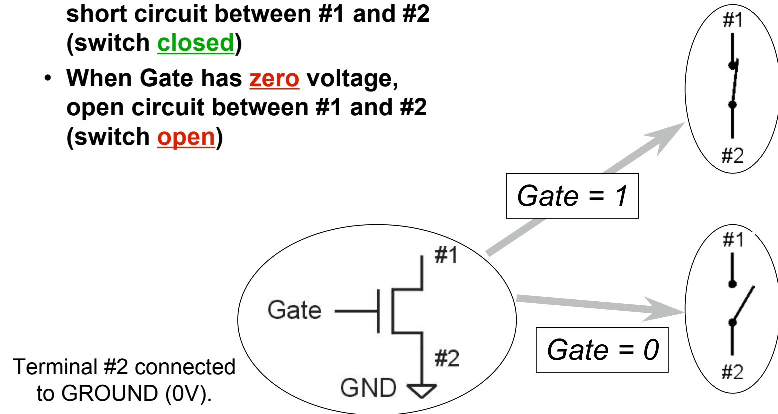
FET: field effect (the mechanism) transistor

- Voltage on gate: current flows source to drain (transistor on)
- No voltage on gate: no current (transistor off)

Recall, two types of MOSFET: n **and** p

N-type MOS Transistor

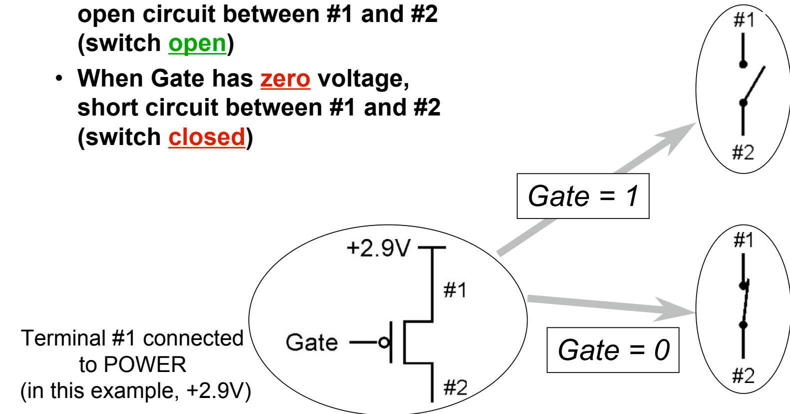
- When Gate has positive voltage, short circuit between #1 and #2 (switch closed)
- When Gate has zero voltage, open circuit between #1 and #2 (switch open)



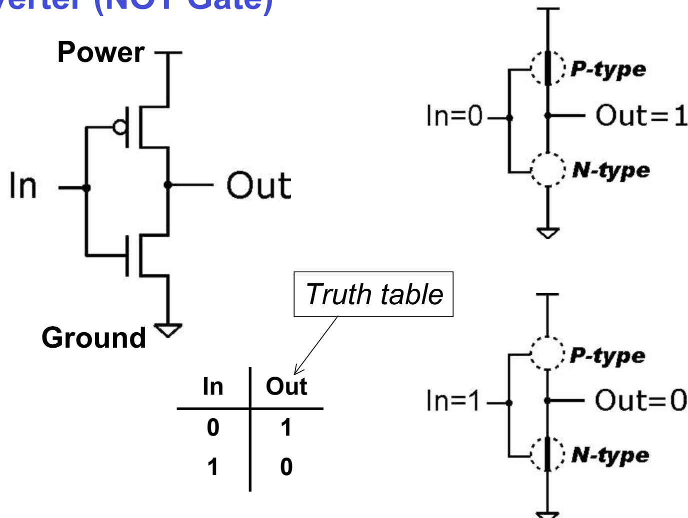
P-type MOS Transistor

P-type is *complementary* to n-type

- When Gate has positive voltage, open circuit between #1 and #2 (switch open)
- When Gate has zero voltage, short circuit between #1 and #2 (switch closed)



Inverter (NOT Gate)



In	Out
0	1
1	0

CMOS Circuit

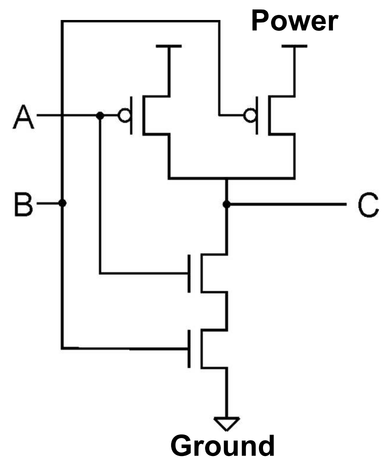
Inverter is an example of Complementary MOS (CMOS)

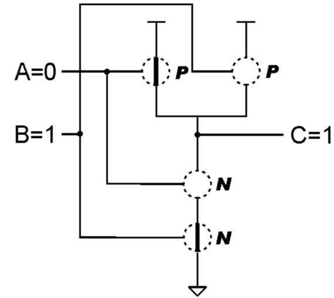
Uses both n-type and p-type MOS transistors

- p-type
  - Attached to POWER (high voltage)
  - Pulls output voltage UP when input is zero
- n-type
  - Attached to GROUND (low voltage)
  - Pulls output voltage DOWN when input is one

For all inputs, make sure that output is either connected to GROUND or to POWER, but not both! (why?)

NAND Gate (NOT-AND)



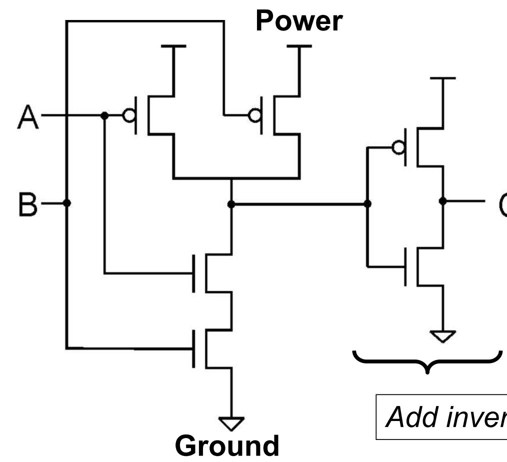


A	B	C
0	0	1
0	1	1
1	0	1
1	1	0

Note: Parallel structure on top, serial on bottom.

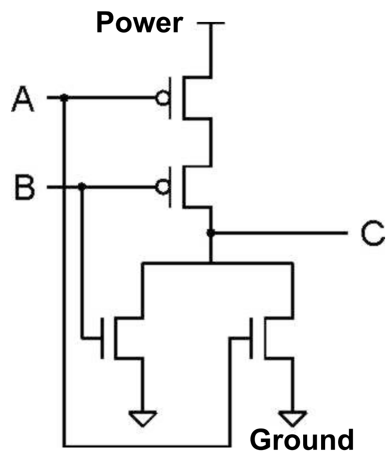
AND Gate

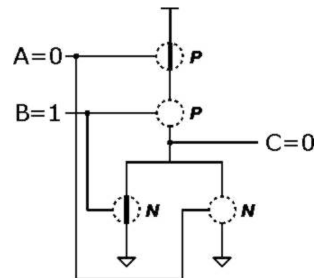
A	B	C
0	0	0
0	1	0
1	0	0
1	1	1



Add inverter to NAND.

NOR Gate (NOT-OR)



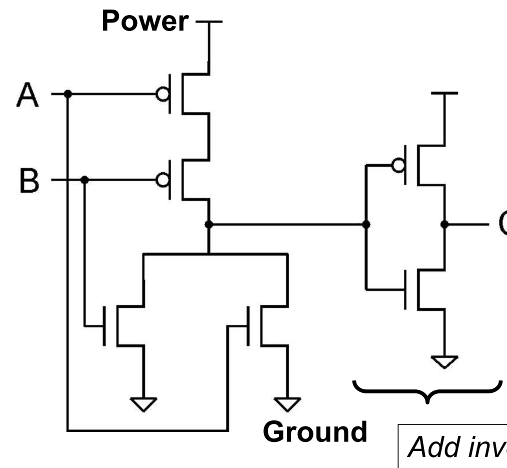


A	B	C
0	0	1
0	1	0
1	0	0
1	1	0

Note: Serial structure on top, parallel on bottom.

OR Gate

A	B	C
0	0	0
0	1	1
1	0	1
1	1	1



Add inverter to NOR.

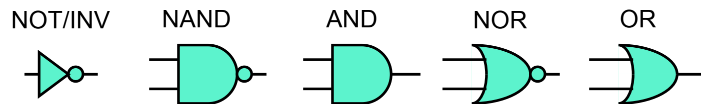
Basic Gates

From Now On… Gates

- Covered transistors mostly so that you know they exist
- Note: "Logic Gate" not related to "Gate" of transistors

Will study implementation in terms of gates

- Circuits that implement Boolean functions



More complicated gates from transistors possible

- XOR, Multiple-input AND-OR-Invert (AOI) gates

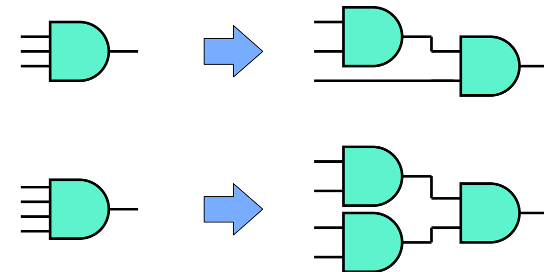
More than 2 Inputs?

AND/OR can take any number of inputs

- AND = 1 if all inputs are 1
- OR = 1 if any input is 1
- Similar for NAND/NOR

Implementation

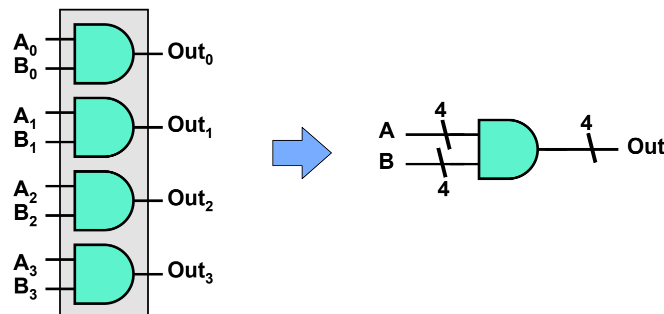
- Multiple two-input gates or single CMOS circuit



Visual Shorthand for Multi-bit Gates

Use a cross-hatch mark to group wires

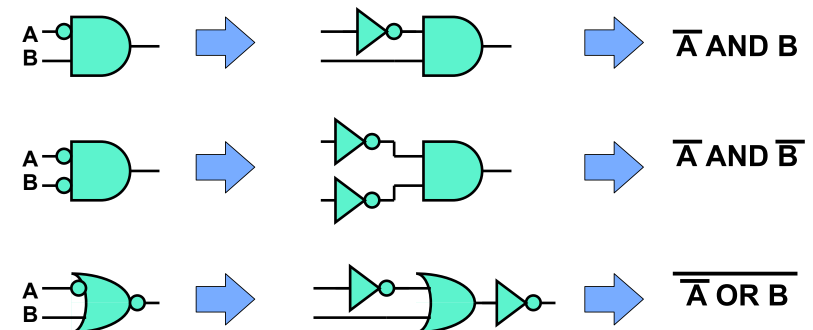
- Example: calculate the AND of a pair of 4-bit numbers
- A_3 is "high-order" or "most-significant" bit
- If "A" is 1000, then A_3 = 1, A_2 = 0, A_1 = 0, A_0 = 0



Shorthand for Inverting Signals

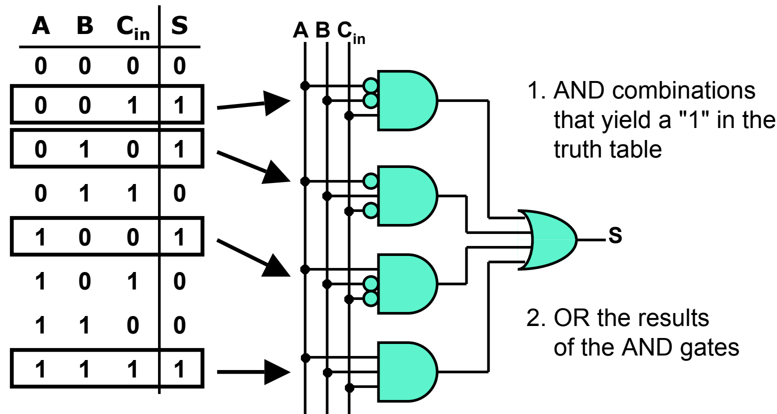
Invert a signal by adding either

- A ○ before/after a gate
- A "bar" over letter



Logical Completeness

AND, OR, NOT can implement ANY truth table



A	B	C_{in}	S
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Logical Completeness via PLAs

Any truth table as a Programmable Logic Array (PLA)

- Traditionally a grid of AND and OR gates
- Configurable by removing wires

Single-output custom PLA (as on previous slide):

- One AND gate per row with "1" in output in truth table
- Maximum number of AND gates: 2^n for n inputs
- One OR gate

Multiple-output custom PLA:

- Build multiple single-output PLAs
- Share AND gates "in common"
- One OR gate per output column in truth table

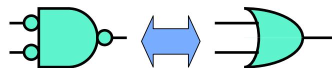
DeMorgan's Law

Converting AND to OR (with some help from NOT)

Consider the following gate:

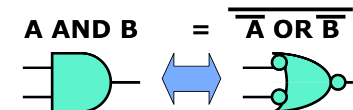
To convert AND to OR (or vice versa), invert inputs and output.

$\overline{\overline{A} \text{ AND } \overline{B}}$ = A OR B

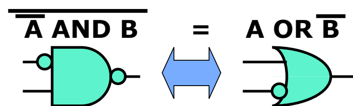


A	B	$\overline{A}$	$\overline{B}$	$\overline{A}$ AND $\overline{B}$	$\overline{\overline{A} \text{ AND } \overline{B}}$
0	0	1	1	1	0
0	1	1	0	0	1
1	0	0	1	0	1
1	1	0	0	0	1

A AND B = $\overline{\overline{A} \text{ OR } \overline{B}}$



$\overline{\overline{A} \text{ AND } B}$ = A OR $\overline{B}$



Why might this be useful?

Summary

MOS transistors: switches to implement logic functions

- n-type: connect to GROUND, turn on (with 1) to pull down to 0
- p-type: connect to POWER, turn on (with 0) to pull up to 1

Basic gates: NOT, NOR, NAND

- Logic functions are usually expressed with AND, OR, and NOT
- Universal: any truth table to simple gates (via a PLA)

DeMorgan's Law

- Convert AND to OR (and vice versa) by inverting inputs/output

Ok, we now have simple logic gates

- Next up: how do we combine them into something useful?

Chapter 3
Digital Logic Structures

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AND, OR, NOT Gates: What Good Are They?

Last time:

- Transistors and gates
- Can implement any logical function using gates (using PLAs)

Today:

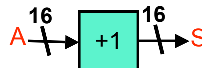
- We'll use gates to create some building blocks of a processor
- One goal: automate binary arithmetic from Chapter 2
- Continuing on our bottom-up journey

Next time:

- Storing bits (memory)
- Circuits with "state"

Incrementer

Let's create a incrementer



- Input: **A** (as a 16-bit 2's complement integer)
- Output: **A+1** (also as a 16-bit 2's complement integer)

Approach #1 (impractical):

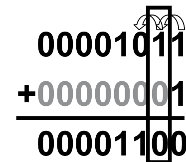
- Use PLA-like techniques to implement circuit
- Problem: 2^{16} or 65536 rows, 16 output columns
- In theory, possible; in practice, intractable

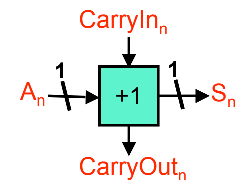
Approach#2 (pragmatic):

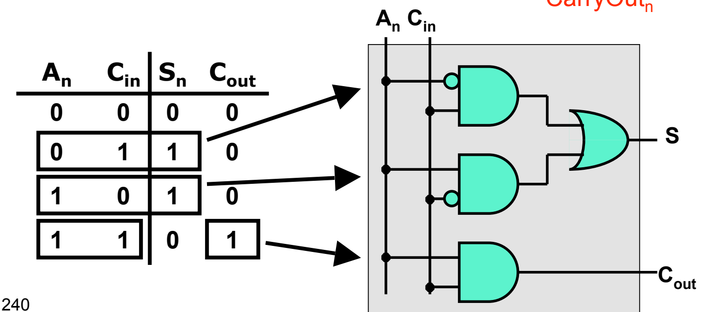
- Create a 1-bit incrementer circuit
- Replicate it 16 times

One-bit Incrementer

Implement a single-column of an incrementer



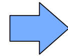


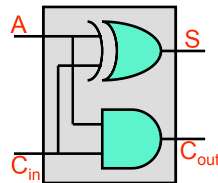


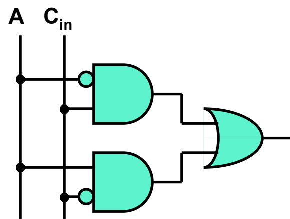
A_n	C_{in}	S_n	C_{out}
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

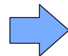
Aside: XOR

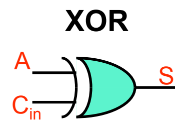
A_n	C_{in}	S_n	C_{out}
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1





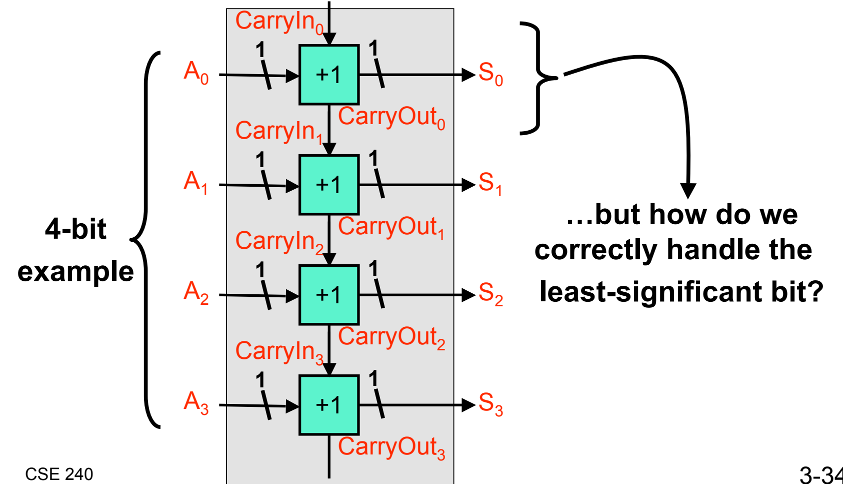






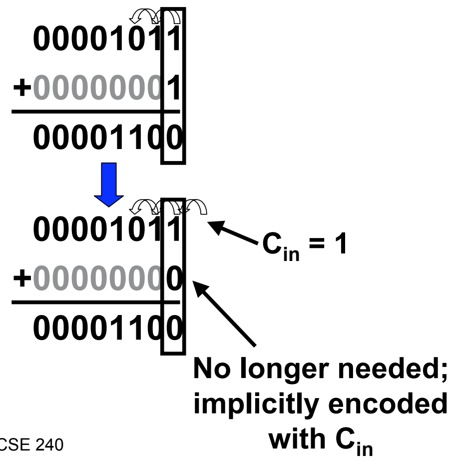
N-bit Incrementer

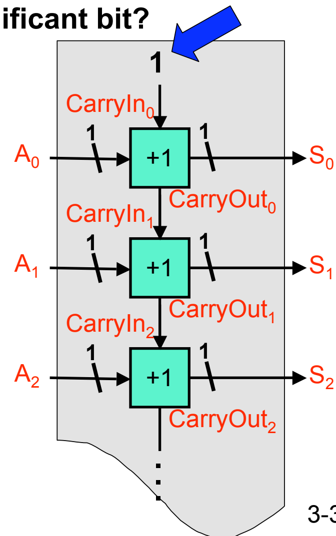
Chain N 1-bit incrementers together



N-bit Incrementer, continued

How do we handle the least-significant bit?

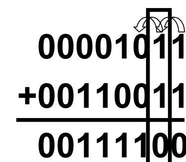


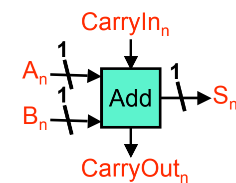


Adder

Conceptually similar to an incrementer

- Build a one-bit slice, replicate *n* times

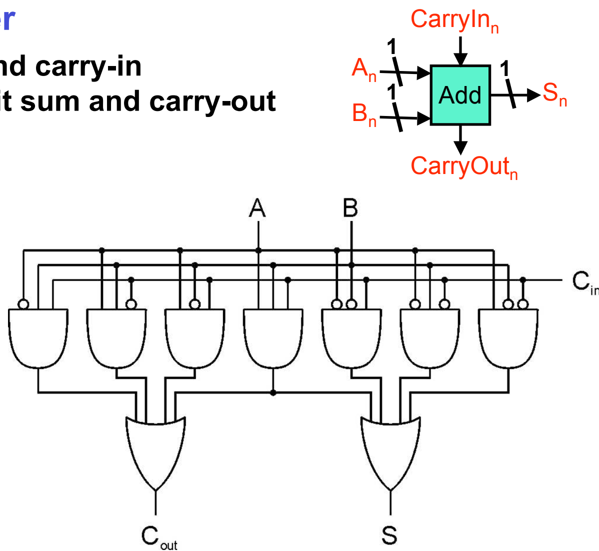




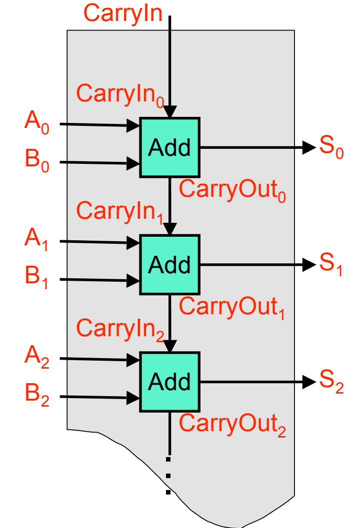
One-bit Adder

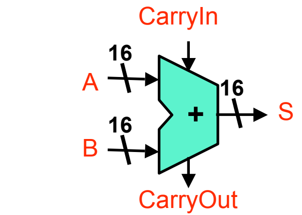
Add two bits and carry-in produce one-bit sum and carry-out

A	B	C_{in}	S	C_{out}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1



N-bit Adder





CarryOut: useful for detecting overflow

CarryIn: assumed to be zero if not present

Aside: Efficient Adders

Full disclosure:

- Our adder: Ripple-carry adder
- No one (sane) actually uses ripple-carry adders
- Why? *way* too slow
- Latency proportional to *n*

We can do better

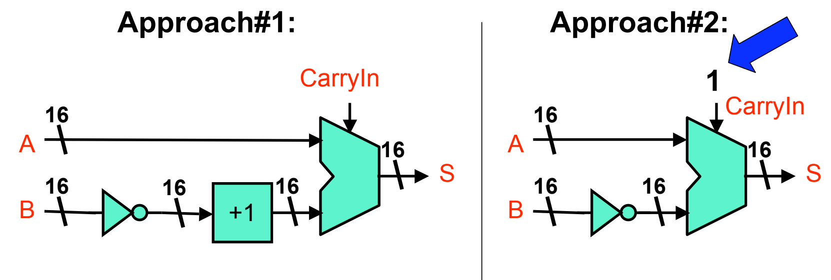
- Many ways to create adders with latency proportional to $log_2(n)$
- In theory: constant latency (build a big PLA)
- In practice: too much hardware, too many high-degree gates
- “Constant factor” matters, too
- More on this topic in CSE371

Subtracter

Build a subtracter from an adder

- Calculate A - B = A + -B
- Negate B
- Recall -B = NOT(B) + 1

Approach#1: **Approach#2:**

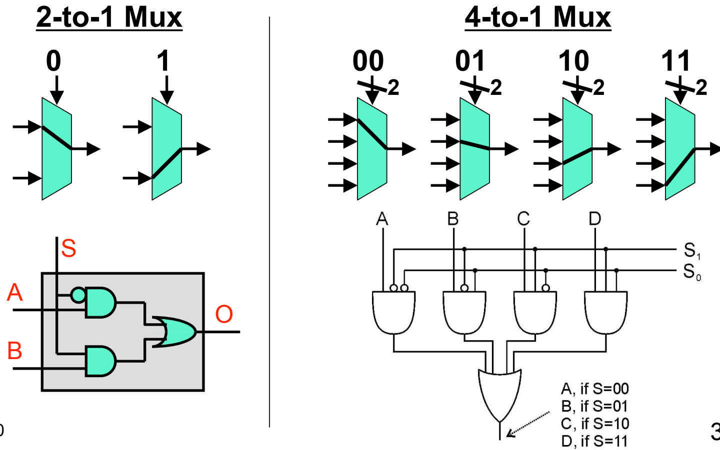


Now, let’s create an adder/subtracter

…But First, The Multiplexer (MUX)

Selector/Chooser of signals

- Multi-way switch



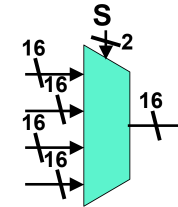
The Multiplexer (MUX)

In general

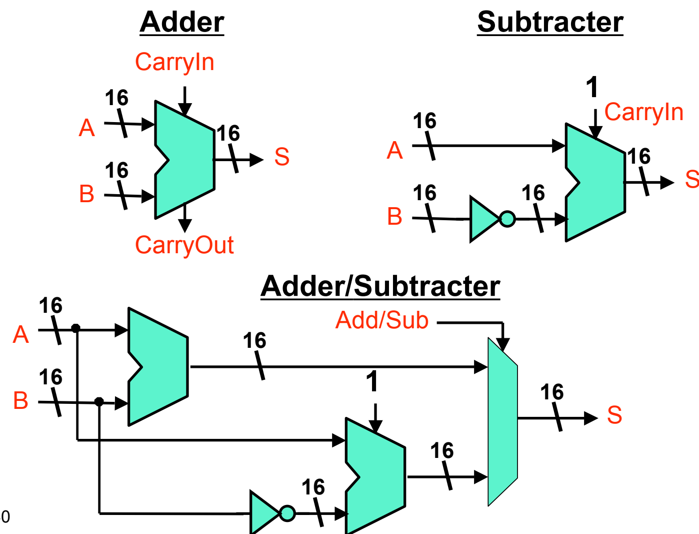
- N select bits chooses from 2^N inputs
- An incredibly useful building block

Multi-bit muxes

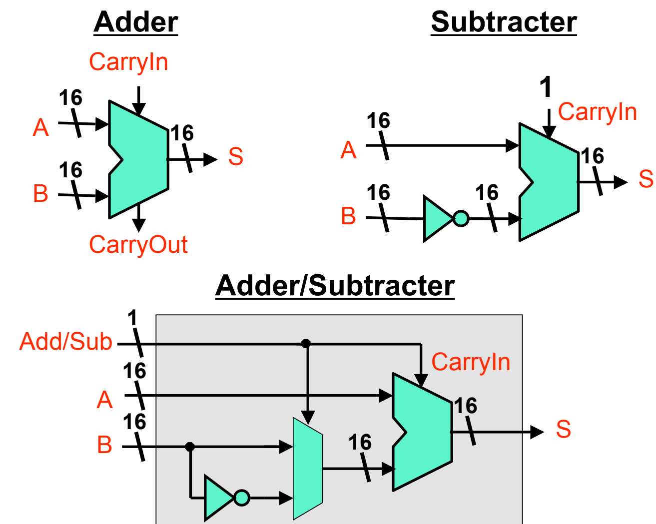
- Can switch an entire "bus" or group of signals
- Switch n-bits with *n* muxes with the same select bits



Adder/Subtracter - Approach #1



Adder/Subtracter - Approach #2



Ok, So We Can Add and Subtract

Other arithmetic operations similar
- **Even floating point operations**

We can calculate; but we can't remember
- **Next time: storage and memory**
- **After that: simple "state machines"**
- **After that: a simple processor**

Remember: readings, quizzes, and homework
- **Homework 2 due Friday**

Chapter 3
Digital Logic Structures

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Combinational vs. Sequential Logic

Combinational Circuit
- **Always gives the same output for a given set of inputs**
  - **For example, adder always generates sum and carry, regardless of previous inputs**

Sequential Circuit
- **Stores information**
- **Output depends on stored information (state) plus input**
  - **Given input might produce different outputs, depending on stored information**
- ***Example:* ticket counter**
  - **Advances when you push the button**
  - **Output depends on previous state**
- **Useful for building "memory" elements and "state machines"**

Storage - Cross-Coupled Inverters

Cross-coupled inverters (INV) gates
- **Holds value Q and Q' (Q' is the same as $\overline{Q}$)**

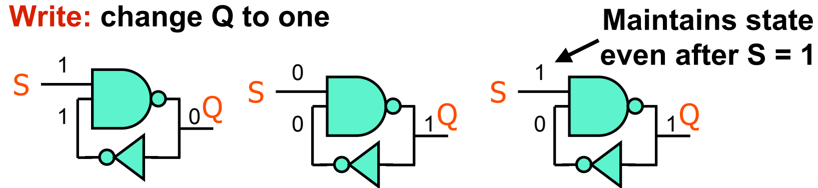
- Read: **get value from either Q or Q'**

Maintains its "state", but how do we change the state?
- Write: **Option #1: put opposite values on Q and Q' simultaneously**
  - **Requires "analog" overdriving of Q and Q'**

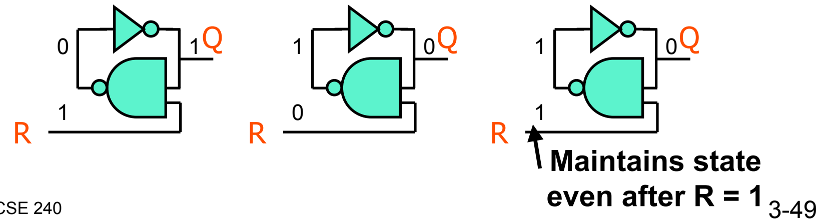
Storage - NANDs

Option #2: "Digital" alternative for changing state

Write: change Q to one



Write: change Q to zero



Storage - Cross-Coupled NANDs (R-S Latch)

Write either a zero or one

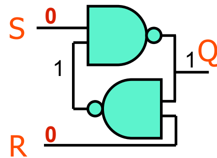
- When S=1 and R=1, "quiescent" state; maintains value

- When S=0 and R=1, state changes to one ("set")
- When S=1 and R=0, state changes to zero ("reset" or "clear")

Storage - Cross-Coupled NANDs (R-S Latch)

What happens with S=0 and R=0?

- Short answer: bad things
- Long answer: value stored will depend on timing on circuit



- Does S or R go to one first?
  - If they change at the same time?
  - Oscillation or meta-stability can result
- Let's make sure this can never happen…

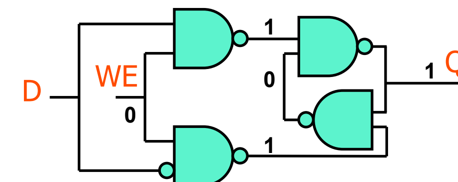
Gated D-Latch

Add logic to an R-S latch

- Create a better interface

Two inputs: D (data) and WE (write enable)

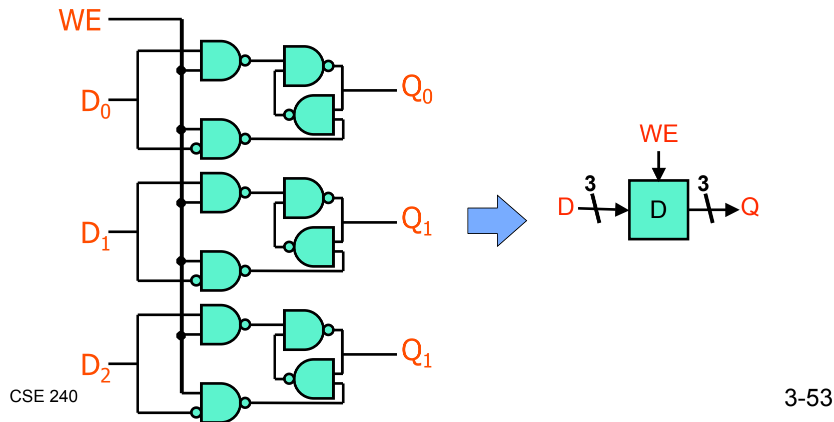
- When WE = 1, latch is set to value of D
  - S = NOT(D), R = D
- When WE = 0, latch continues to hold previous value
  - S = R = 1
- Does not allow S=0, R=0 case to occur



Register

A register stores a multi-bit value

- **A collection of D-latches, controlled by a common WE**
- **When WE=1, n-bit value D is written to register**



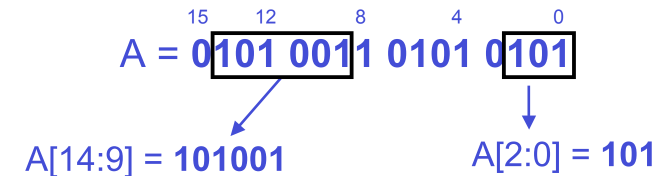
Aside: More on Representing Multi-bit Values

Number bits from right (0) to left (n-1)

- **Just a convention -- could be left to right, but must be *consistent***

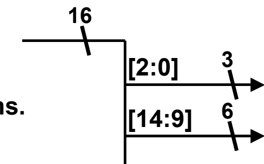
Use brackets to denote range:

D[l:r] **denotes bit** l **to bit** r**, from *left* to *right***

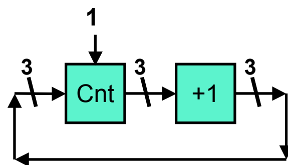


May also see A<14:9>

- **Especially in hardware block diagrams.**



Let's Try to Build a Counter



How quickly will this count?

- **Timing dependent**

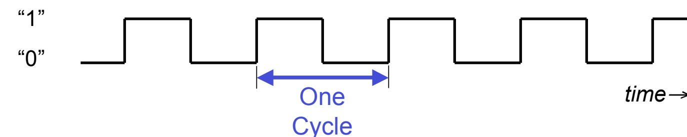
Will it even work?

- **Probably not**
- **D-latches are "transparent"**
  - **Allows next input to immediately flow to output**
  - **Outputs will never be "stable"**

What's Missing? The Clock

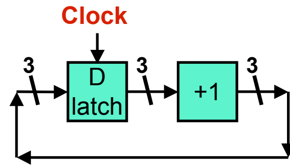
A clock controls when registers are "updated"

- **Oscillating global signal with fixed period**
- **Typical clock frequencies today: a couple of gigahertz**



- **Corresponds to <1 nanosecond between one rise and the next**
- **Generated on-chip by special circuitry (for example, oscillating ring of inverters)**

Let's Try Again: a Counter

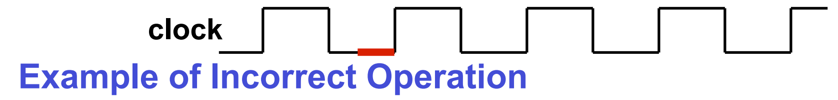


Solves half the problem

- Controls the rate of updates

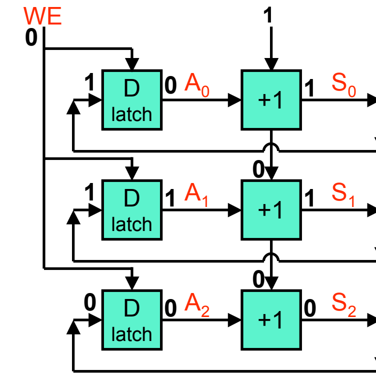
Remaining problem

- When clock=1, same problem
- D-latches are still transparent



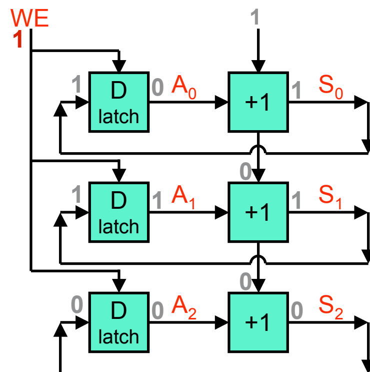
Example of Incorrect Operation

Initial state: 010_2



Example of Incorrect Operation

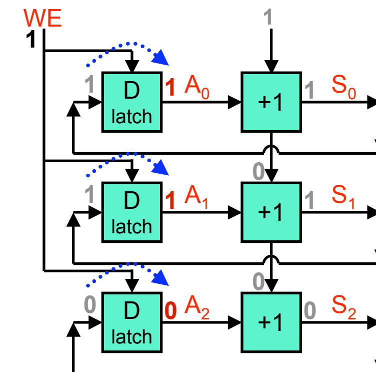
Set WE (Write Enable) to 1



Example of Incorrect Operation

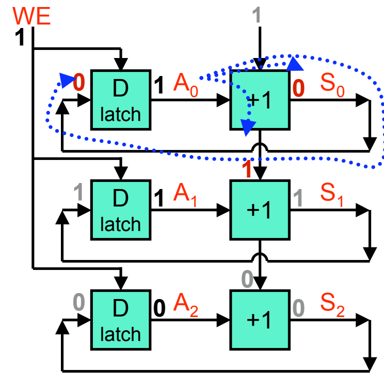
D latches write new value: 011_2

Goal: 100_2



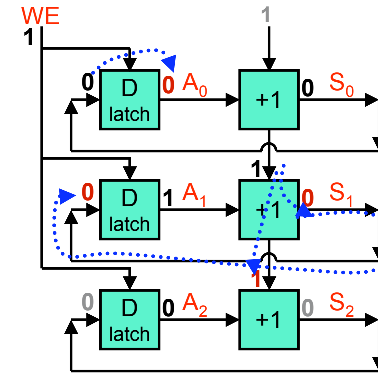
Example of Incorrect Operation

Incrementer calculates 1st bit



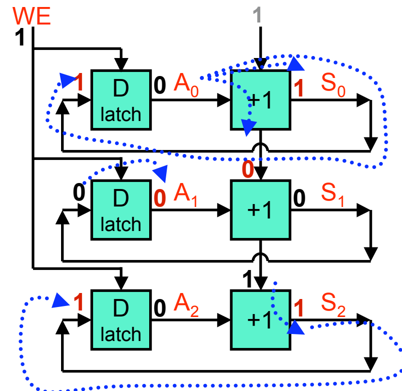
Example of Incorrect Operation

Incrementer calculates 2nd bit, 1st bit latched



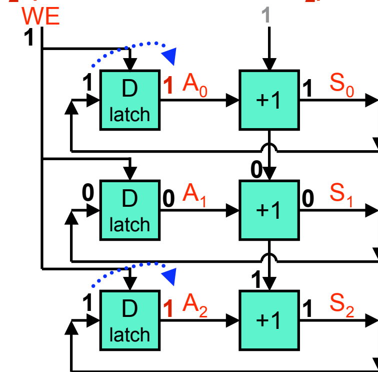
Example of Incorrect Operation

Incrementer calculates 3rd bit, 2nd bit latched,
1st bit re-calculated



Example of Incorrect Operation

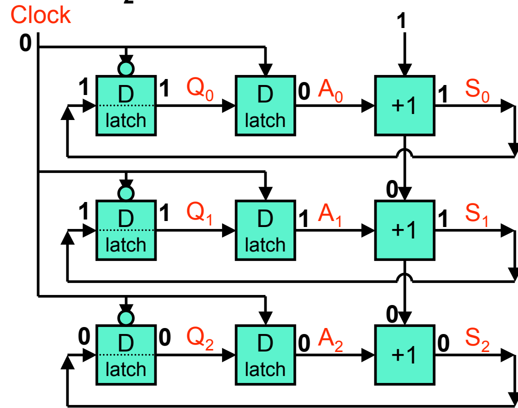
1st and 3rd bits latched
value is: 101_2 (not the desired 100_2)



Correct Operation

Additional D-latches, WE is NOT(Clock) and Clock

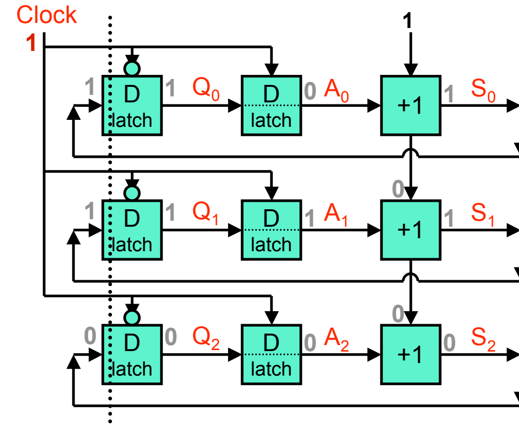
Initial state: 010_2



clock

Correct Operation

Clock switches to 1, 2nd latch WE = 1

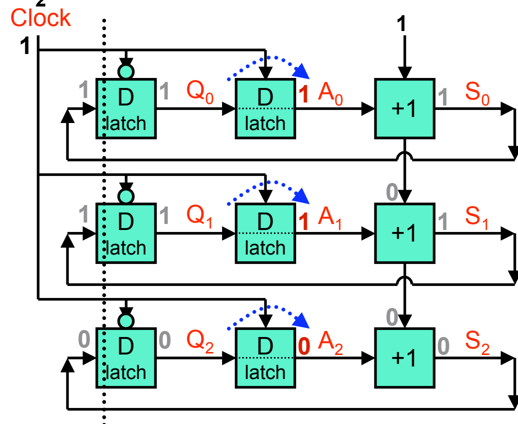


clock

Correct Operation

D latches write new value: 011_2

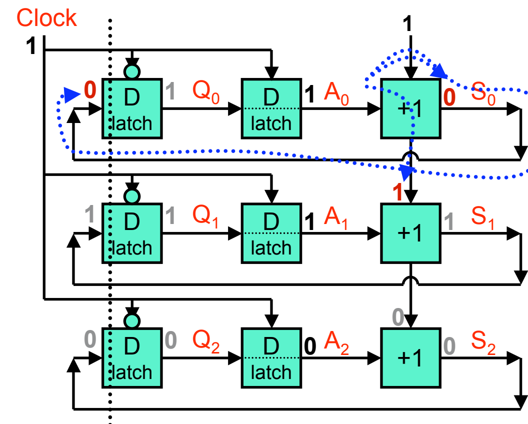
Goal: 100_2



clock

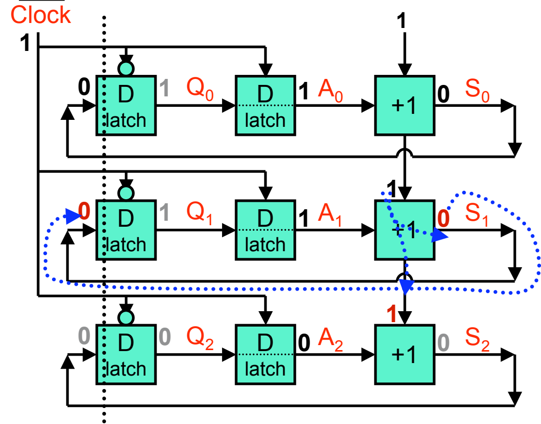
Correct Operation

Incrementer begins calculation



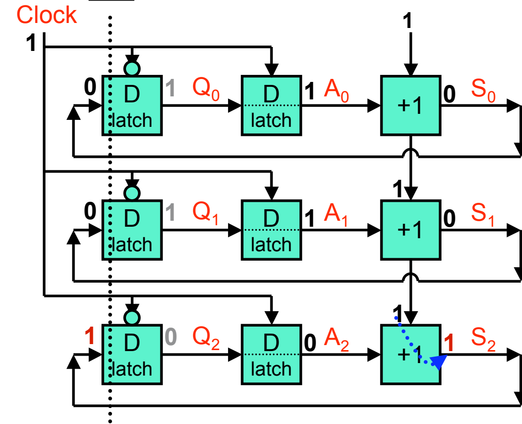
Correct Operation

Incrementer calculates 2nd bit,
First bit <u>not</u> written to latch



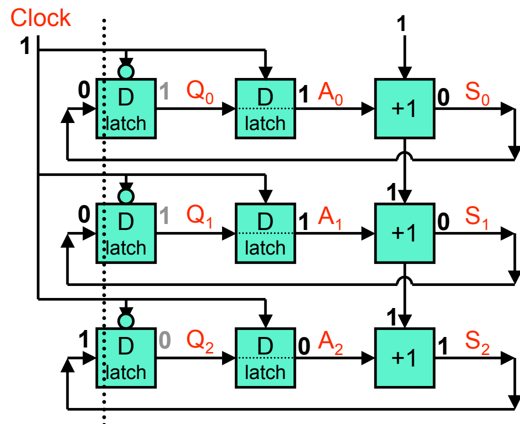
Correct Operation

Incrementer calculates 3rd bit,
1st, 2nd bits <u>not</u> written to latch



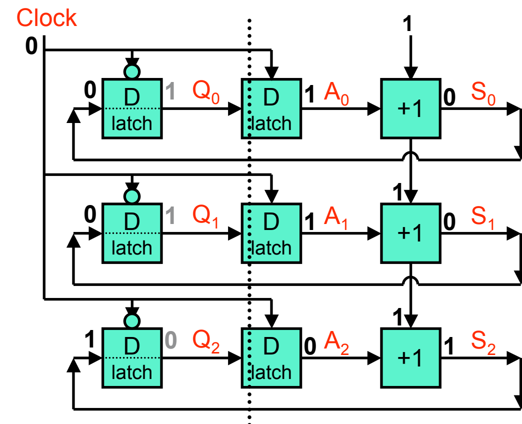
Correct Operation

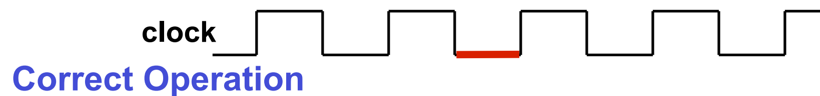
Correct value ready to latch (100_2), circuit <u>quiescent</u>



Correct Operation

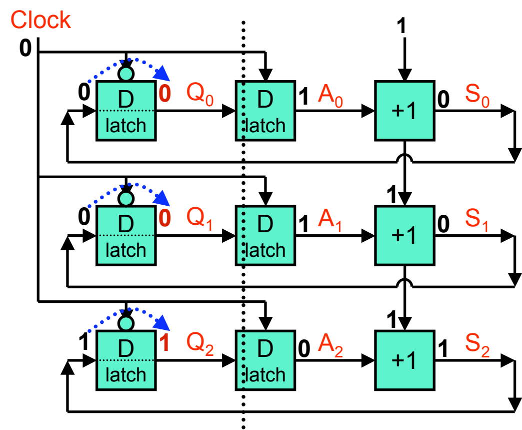
Clock changes to 0

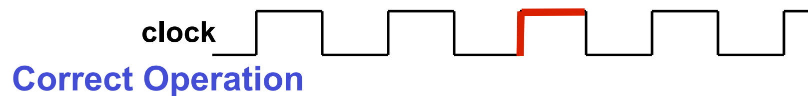




Correct Operation

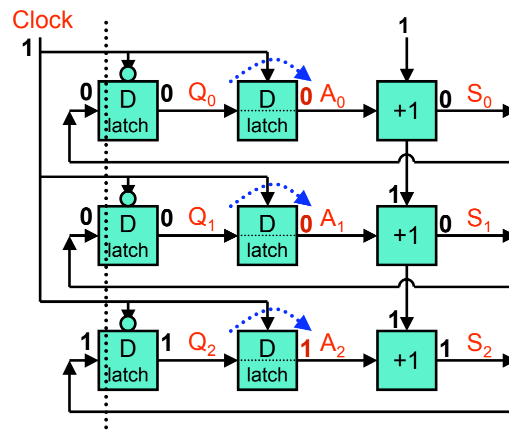
2nd set of latches write correct value, circuit quiescent





Correct Operation

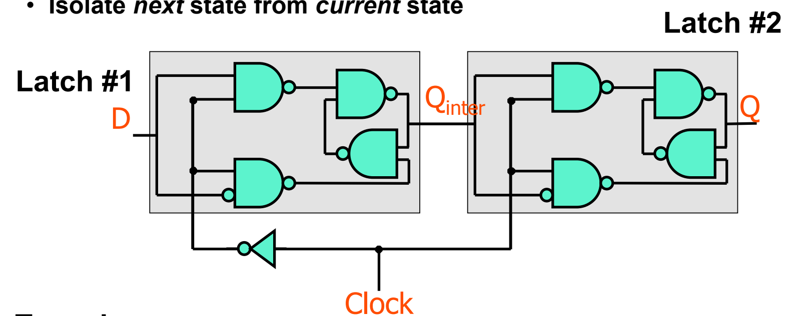
Clock changes back to 1, next increment begins



D Flip-Flop (or master-slave flip-flop)

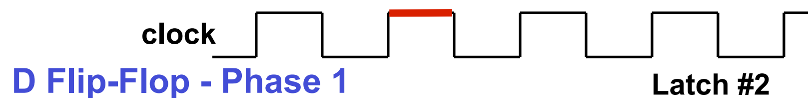
D Flip-Flop is a pair of D latches

- **Stupid name, but it stuck**
- **Isolate *next* state from *current* state**

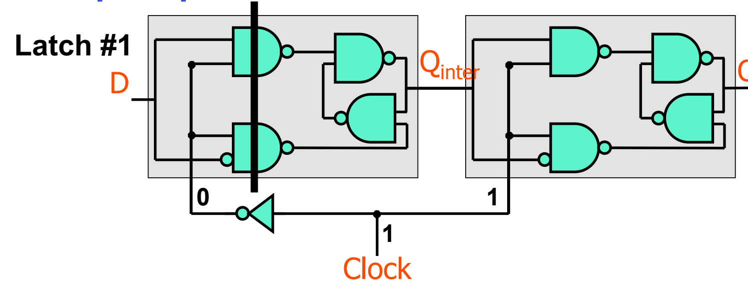


Two phases:

- **Clock = 1, Clock = 0**



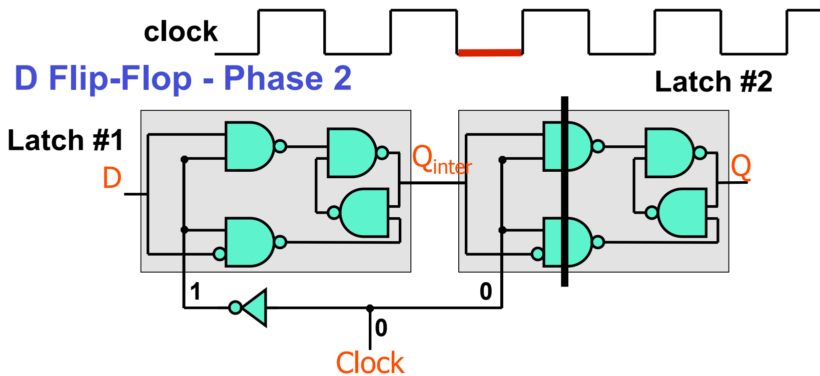
D Flip-Flop - Phase 1



Phase 1

- **Clock = 1**
- **Latch 1: writing disabled (output is stable Q_{inter})**
- **Latch 2: writing enabled ($Q = Q_{inter}$)**

D Flip-Flop - Phase 2



Phase 2

- Clock = 0
- Latch 1: writing enabled (Q_{inter} = D)
- Latch 2: writing disabled (output is stable Q)

Back to Phase 1

- Q becomes Q_{inter}

Latches & Flip-Flops

Latches

- "level triggered" (high or low)
- "transparent"

Flip-Flops

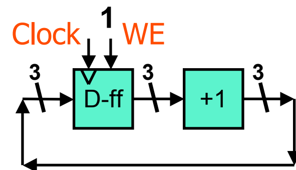
- "edge triggered" (rising/positive edge or falling/negative edge)
- "non-transparent" or "opaque" or just "latch" (!!!)

Flip-Flops have WE (write enable) signals, too

- Uses a gate to suppress the rising (or falling) edge of clock
- Once internalized in FF, no need to manipulate clock with logic
- Otherwise manipulating clock with logic usually a bad idea™

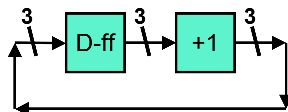
Working Counter

Use a clocked register (made of D flip-flops)



More simply

- If WE = 1 assumed if WE is not present



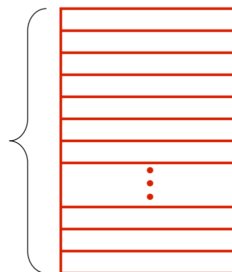
Use WE input for conditional counter (stop watch)

Memory

Now that we know how to store bits, we can build a memory – a logical *k by m* array of stored bits

Address Space:
number of locations
(usually a power of 2)

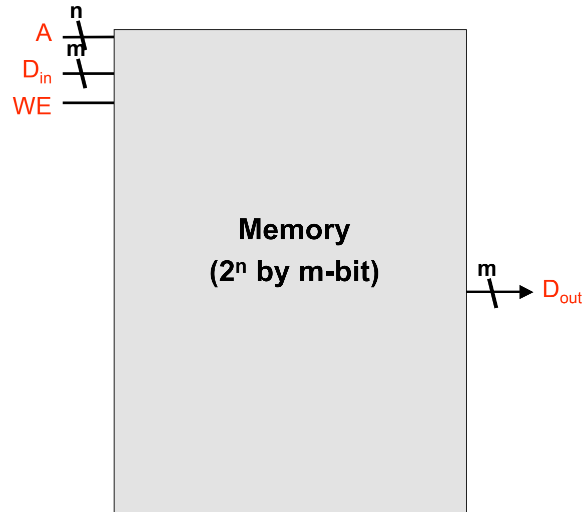
$k = 2^n$ locations



Addressability:
number of bits per location
(e.g., byte-addressable)

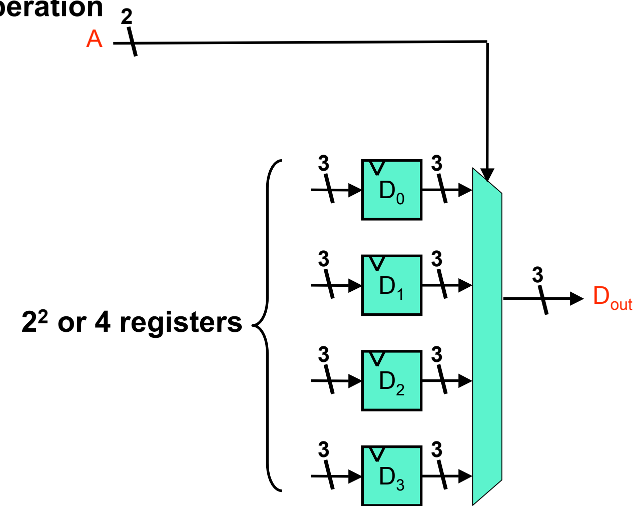
***m* bits**

Memory Interface



2^2 by 3-bit memory

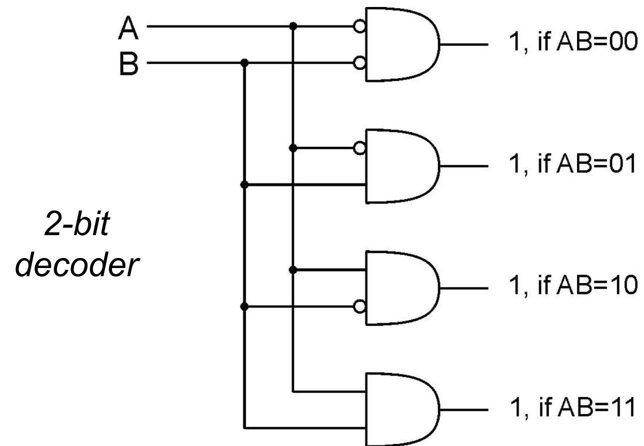
Read operation



The Decoder

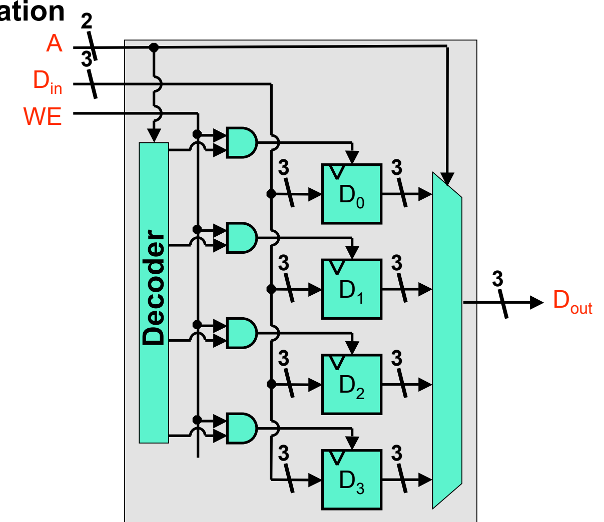
***n* inputs, 2^n outputs**

- **Exactly one output is 1 for each possible input pattern**



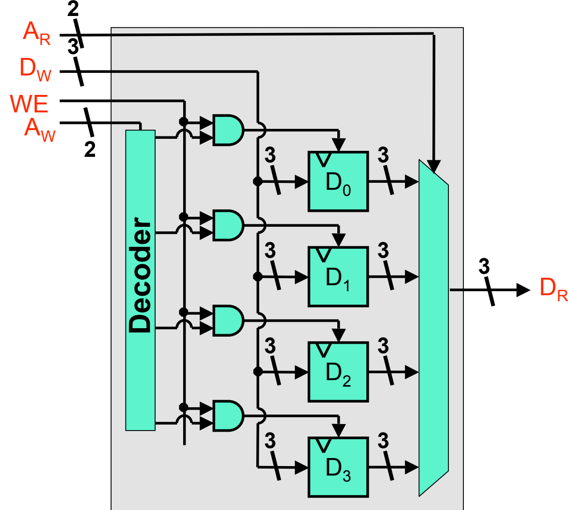
2^2 by 3-bit memory

Write operation

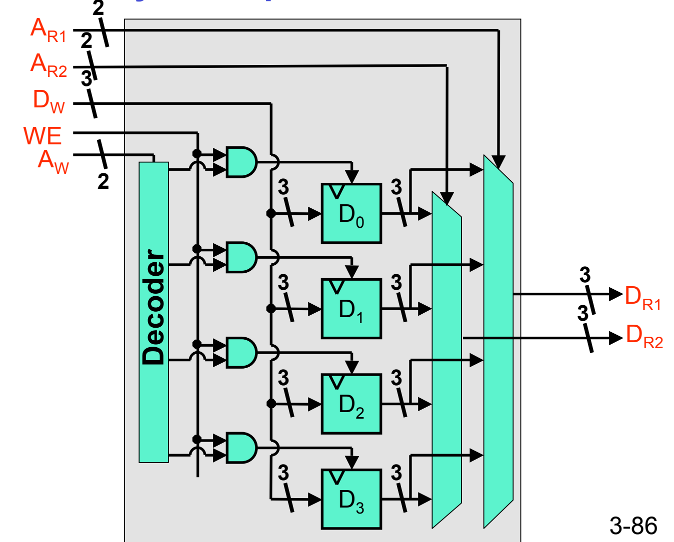


2^2 by 3-bit memory - Multiple "Ports"

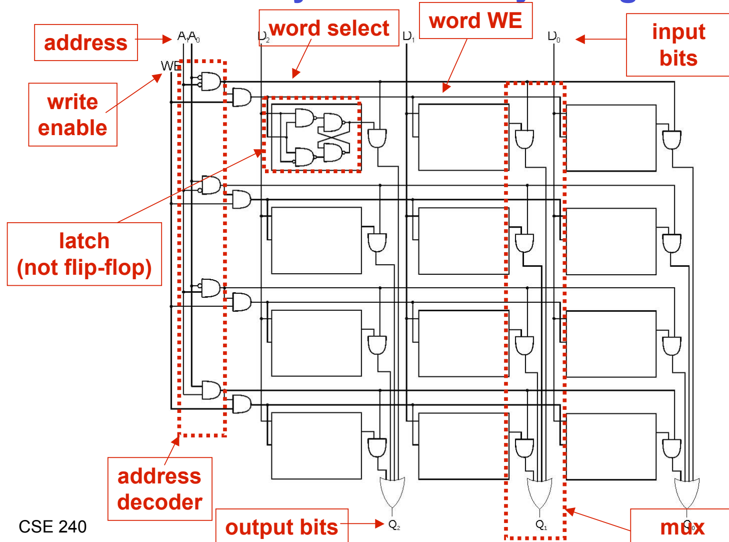
Independent Read/Write



2^2 by 3-bit memory - Multiple Read Ports



An Efficient 2^2 by 3-bit Memory - Single Port



More Memory Details

This is still not the way actual memory is implemented

- **Real memory: fewer transistors, denser, relies on analog properties**

But the logical structure is similar

- **Address decoder**
- **Word select line, word write enable**
- **Bit line**

Two basic kinds of RAM (Random Access Memory)

Static RAM (SRAM) - 6 transistors per bit

- **Fast, maintains data as long as power applied**

Dynamic RAM (DRAM) - 1 transistor per bit

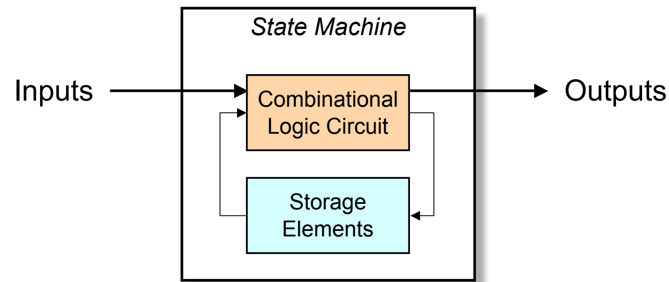
- **Denser but slower, destructive read, bit storage decays – must be periodically refreshed (like a leaky balloon)**

Also, non-volatile memories: ROM, PROM, flash, …

State Machine

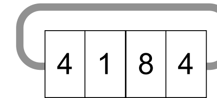
Another type of sequential circuit

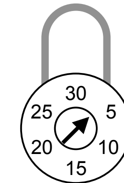
- Combines combinational logic with storage
- "Remembers" state, and changes output (and state) based on inputs and current state



Combinational vs. Sequential

Two types of "combination" locks





Combinational
Success depends only on the values, not the order in which they are set.

Sequential
Success depends on the sequence of values (e.g, R-13, L-22, R-3).

State

The state of system is snapshot of all relevant elements of system at moment snapshot is taken

Examples

- The state of a basketball game can be represented by the scoreboard
  - Number of points, time remaining, possession, *etc.*
- The state of a tic-tac-toe game can be represented by the placement of X's and O's on the board (and turn)

State of Sequential Lock

Our lock example has four different states, labeled A-D:

**A: The lock is not open,
and no relevant operations have been performed**

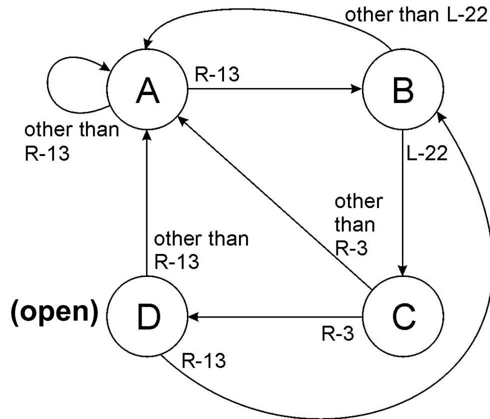
**B: The lock is not open,
and the user has completed the R-13 operation**

**C: The lock is not open,
and the user has completed R-13, followed by L-22**

D: The lock is open

Sequential Lock State Diagram

Shows states and actions that cause a transition between states



Finite State Machine

A description of a system with the following components:

1. **A finite number of states**
2. **A finite number of external inputs**
3. **A finite number of external outputs**
4. **An explicit specification of all state transitions**
5. **An explicit specification of what determines each external output value**

Often described by a state diagram

- **Inputs trigger state transitions**
- **Outputs are associated with each state (or with each transition)**

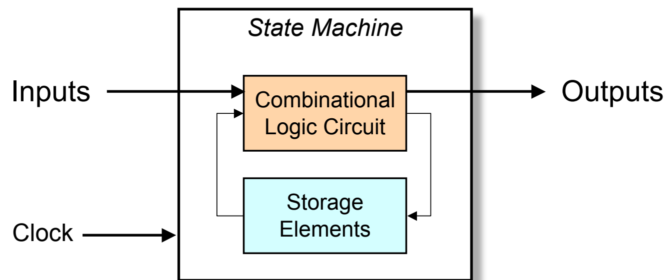
Implementing a Finite State Machine

Combinational logic

- **Determine outputs and next state.**

Storage elements

- **Maintain state representation.**



Storage

Master-slave flip-flop stores one state bit

Number of storage elements (flip-flops)

- **Determined by number of states (and representation of states)**

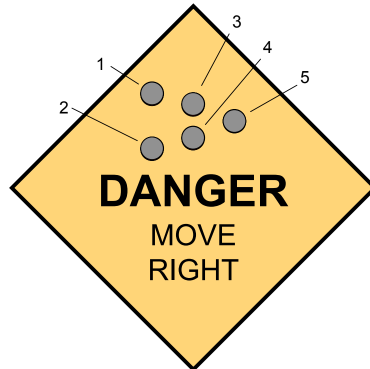
Examples

- **Sequential lock**
  - **Four states – two bits**
- **Basketball scoreboard**
  - **8 bits for each score, 5 bits for minutes, 6 bits for seconds, 1 bit for possession arrow, 1 bit for half, …**

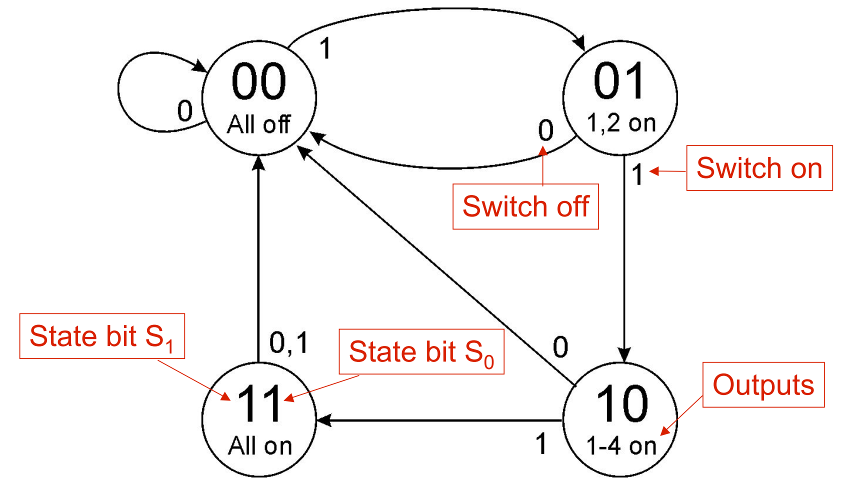
Complete Example

A blinking traffic sign

- **No lights on**
- **1 & 2 on**
- **1, 2, 3, & 4 on**
- **1, 2, 3, 4, & 5 on**
- **(repeat as long as switch is turned on)**

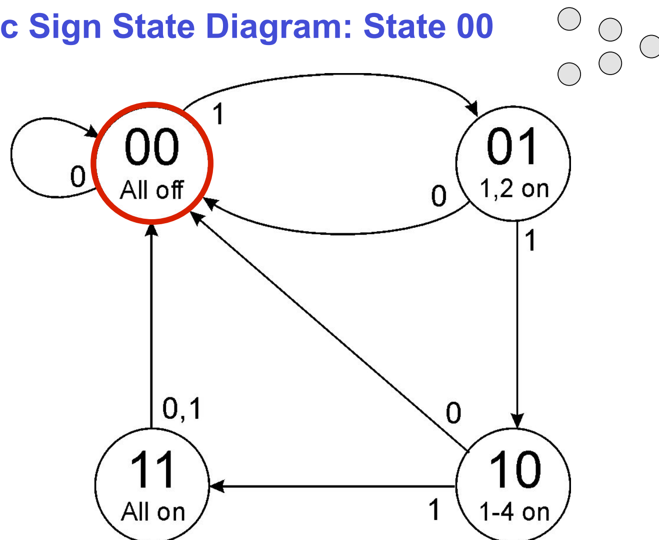


Traffic Sign State Diagram



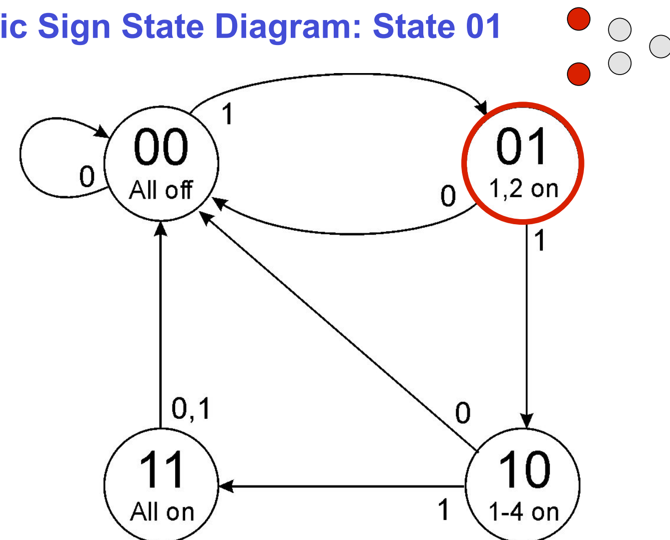
Transition on each clock cycle.

Traffic Sign State Diagram: State 00



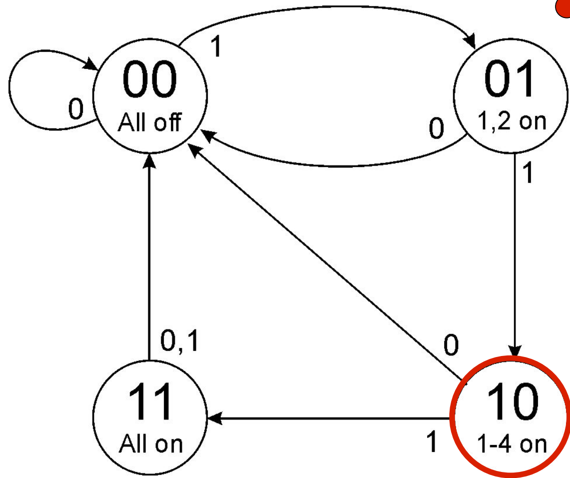
Transition on each clock cycle.

Traffic Sign State Diagram: State 01



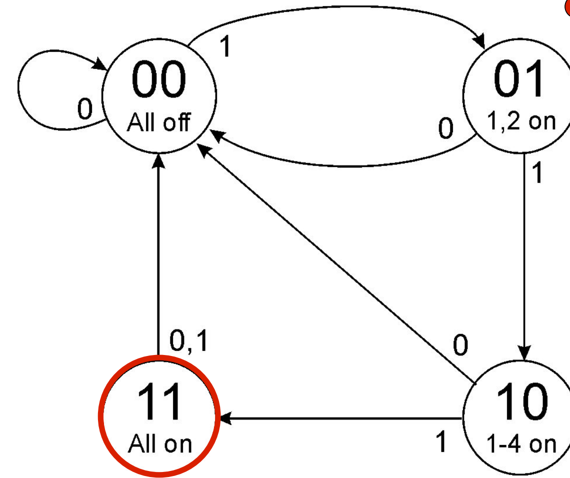
Transition on each clock cycle.

Traffic Sign State Diagram: State 10



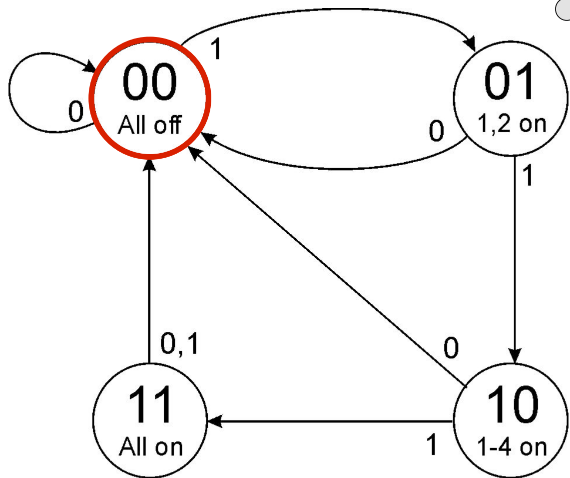
Transition on each clock cycle.

Traffic Sign State Diagram: State 11



Transition on each clock cycle.

Traffic Sign State Diagram: State 00



Transition on each clock cycle.

Traffic Sign Truth Tables

Outputs
(depend only on state: S_1S_0)

Z: Lights 1 and 2; Y: Lights 3 and 4; X: Light 5

S_1	S_0	Z	Y	X
0	0	0	0	0
0	1	1	0	0
1	0	1	1	0
1	1	1	1	1

Next State: $S_1'S_0'$
(depend on state and input)

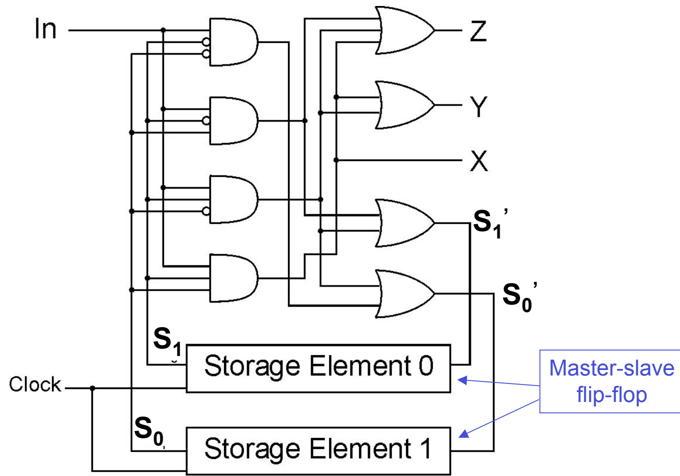
In: Switch

In	S_1	S_0	S_1'	S_0'
0	X	X	0	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	1
1	1	1	0	0

Don't care (1 or 0)

Whenever In=0, next state is 00.

Traffic Sign Logic

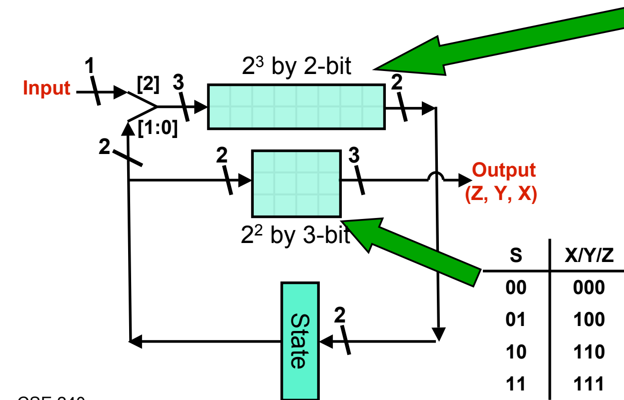


Programmable State Machines

What if we want to change the pattern of the sign?

- An alternative state machine implementation
- Use a memory indexed by state number

In/S_1/S_0	S
000	00
001	00
010	00
011	00
100	01
101	10
110	11
111	00



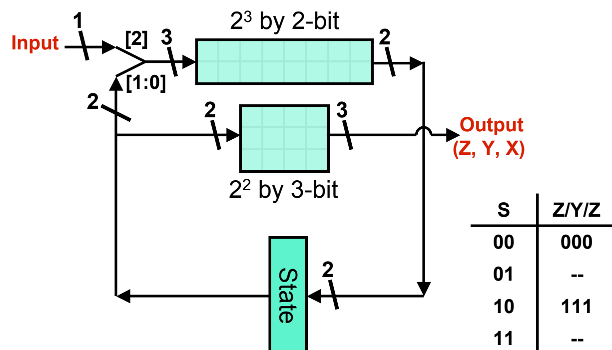
S	X/Y/Z
00	000
01	100
10	110
11	111

Programmable State Machines

Change to a two-state pattern:

- All off
- All on

State: 00 State: 10 State: 00



S	Z/Y/Z
00	000
01	--
10	111
11	--

In/S_1/S_0	S
000	00
001	--
010	00
011	--
100	10
101	--
110	00
111	--

From Logic to Data Path

The data path of a computer is all the logic used to process information.

- See the data path of the LC-3 on next slide

Combinational Logic

- Decoders -- convert instructions into control signals
- Multiplexers -- select inputs and outputs
- ALU (Arithmetic and Logic Unit) -- operations on data

Sequential Logic

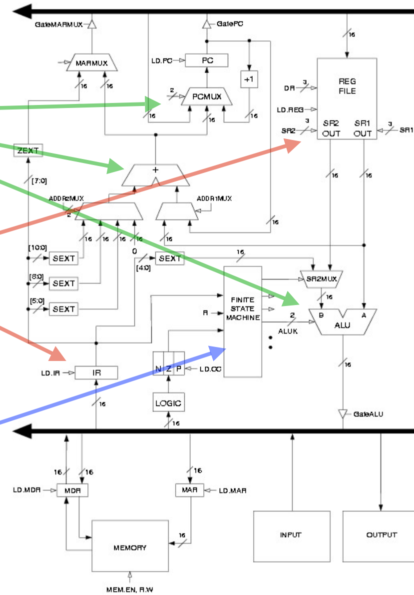
- State machine -- coordinate control signals and data movement
- Registers and latches -- storage elements

LC-3 Data Path

Combinational Logic

Storage

State Machine



Looking Forward…

We’ve touched on basic digital logic

- Transistors
- Gates
- Storage (latches, flip-flops, memory)
- State machines

Built some simple circuits

- Incrementer, adder, subtracter, adder/subtracter
- Counter (consisting of register and incrementer)
- Hard-coded traffic sign state machine
- Programmable traffic sign state machine

Up next: a computer as a (simple?) state machine

Next Time

Topic

- The von Neumann Model

Readings

- Chapter 4.0 - 4.2

Online quiz

- You know the drill!