Segmentation by Weighted Aggregation

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Hierarchical Adaptive Texture Segmentation

or

Segmentation by Weighted Aggregation (SWA)

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Image Segmentation

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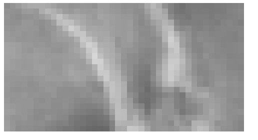
Local Uncertainty



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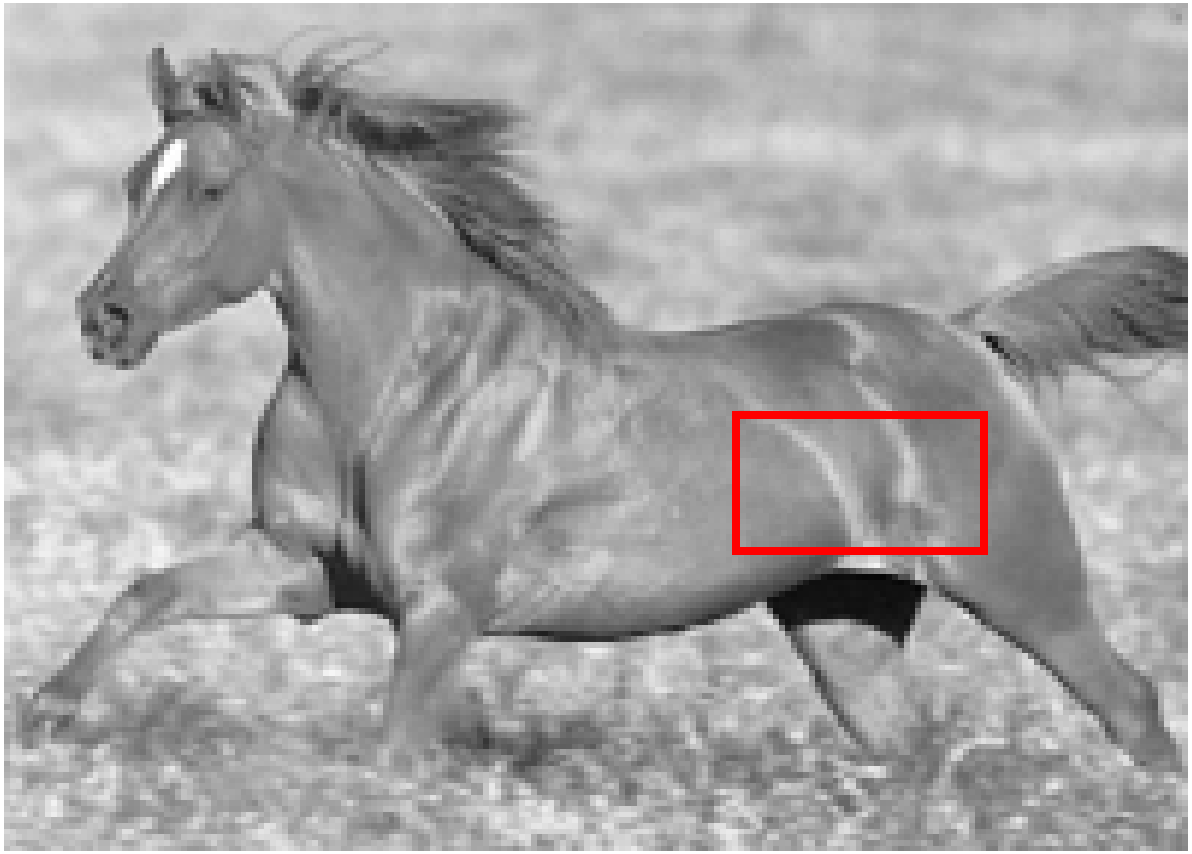
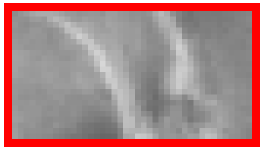
Global Certainty





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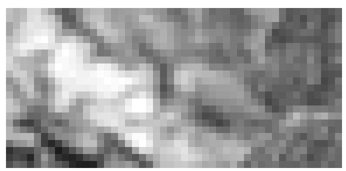
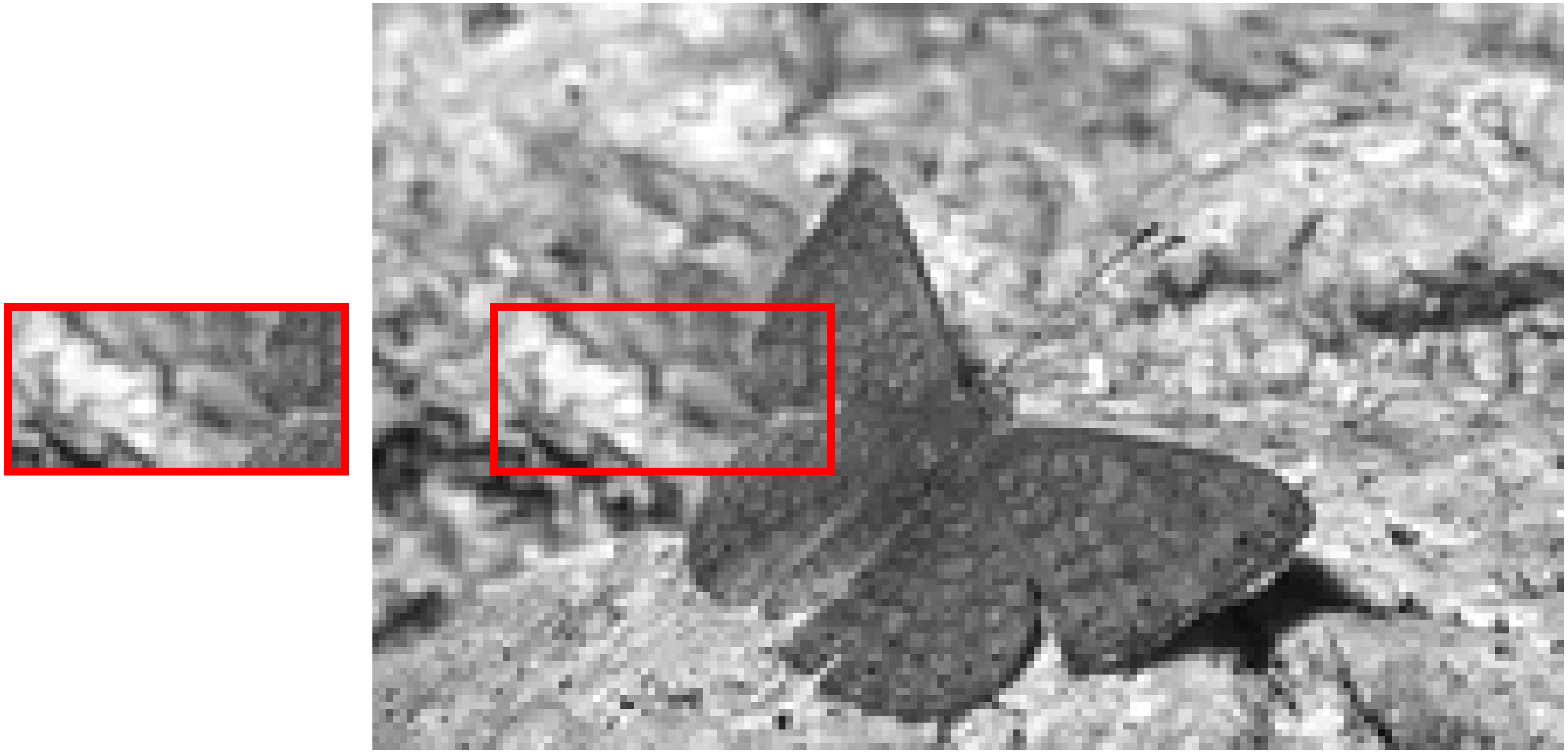
Local Uncertainty



Global Certainty



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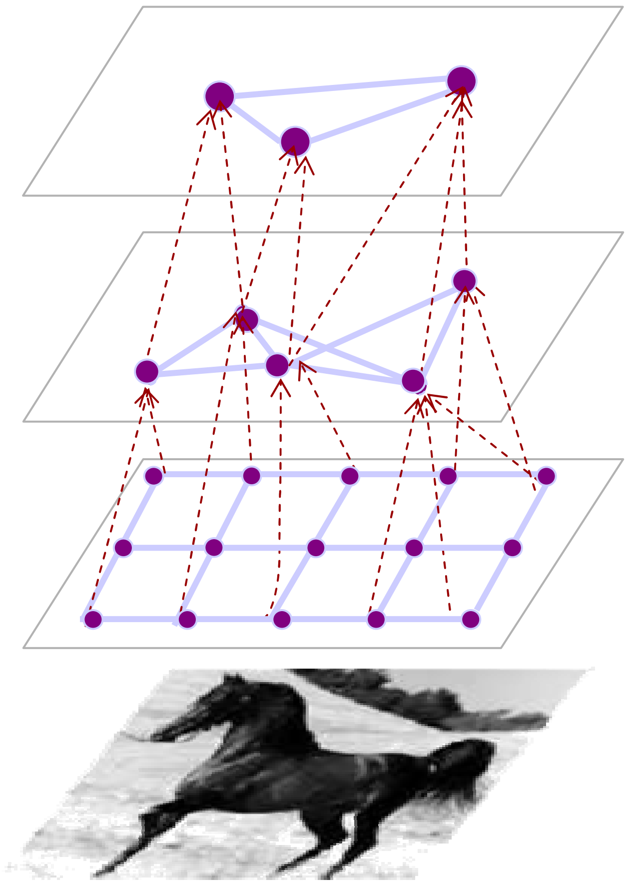
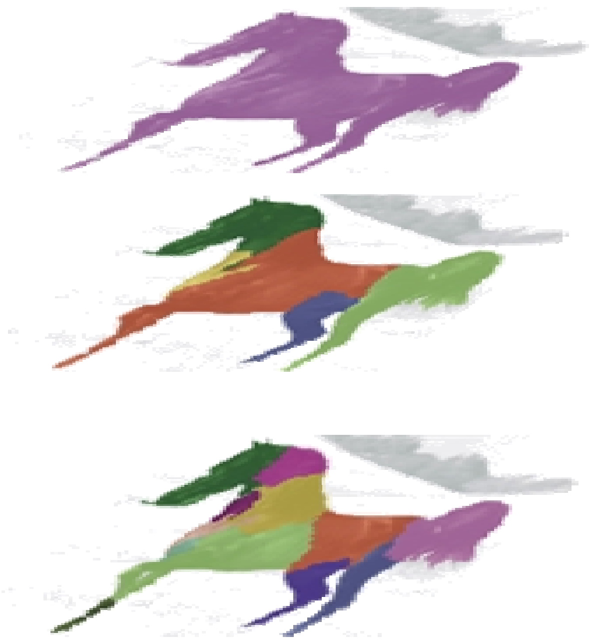
Hierarchy in SWA





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Segmentation by Weighted Aggregation

A multiscale algorithm:

- Optimizes a global measure
- Returns a full hierarchy of segments
- Linear complexity
- Combines multiscale measurements:
  - Texture
  - Boundary integrity

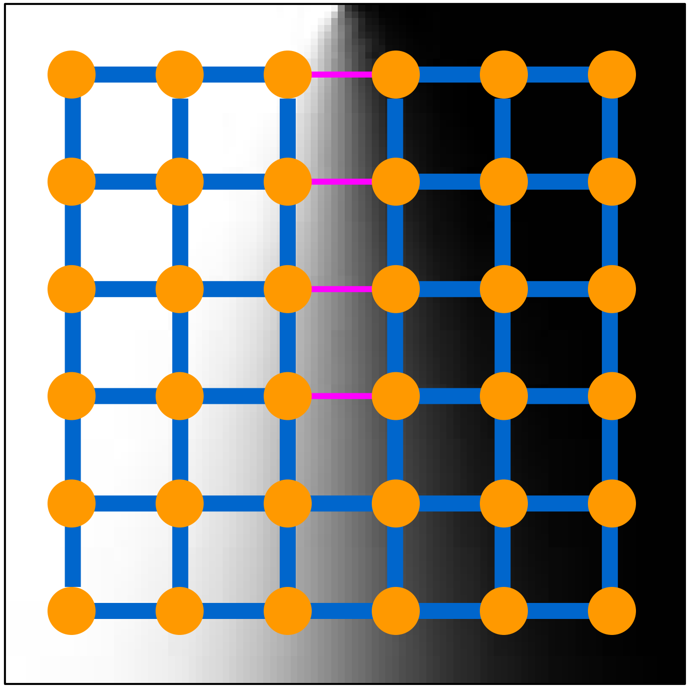
The Pixel Graph

Couplings $\{w_{ij}\}$

Reflect intensity similarity

Low contrast – strong coupling

High contrast – weak coupling



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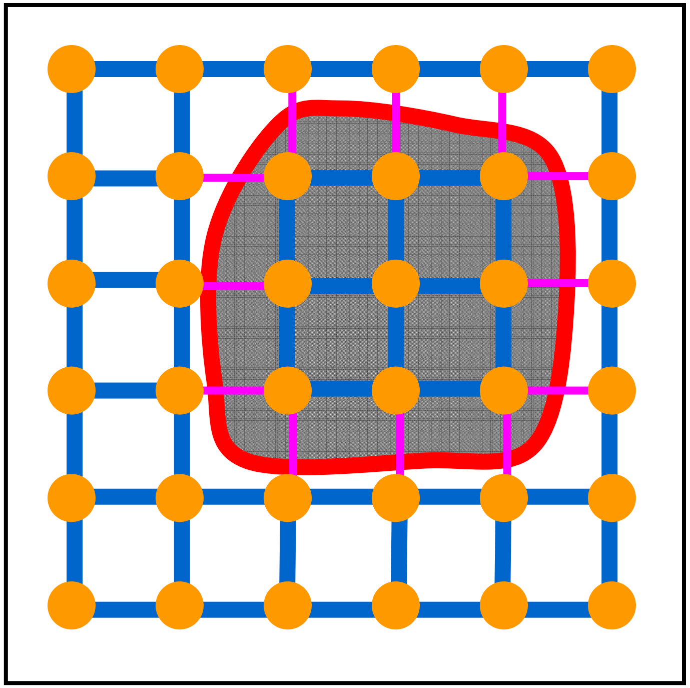
Normalized-Cut Measure

$$u_i = \begin{cases} 1 & i \in S \\ 0 & i \notin S \end{cases}$$



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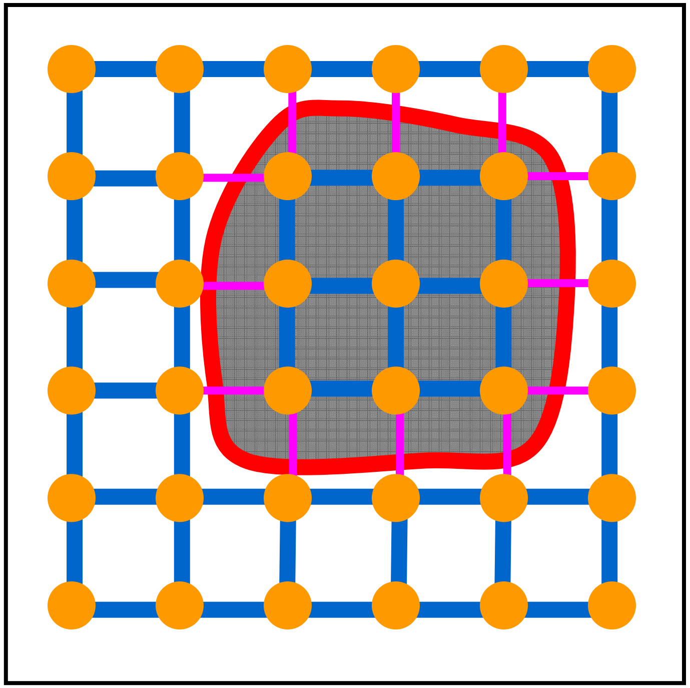
Normalized-Cut Measure

$$E(S) = \sum_{i \neq j} w_{ij} (u_i - u_j)^2$$

$$u_i = \begin{cases} 1 & i \in S \\ 0 & i \notin S \end{cases}$$



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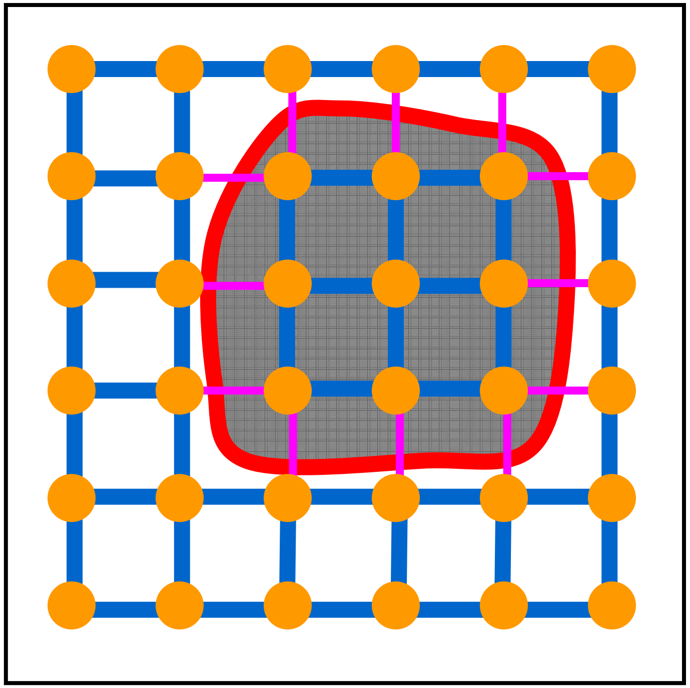

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Normalized-Cut Measure

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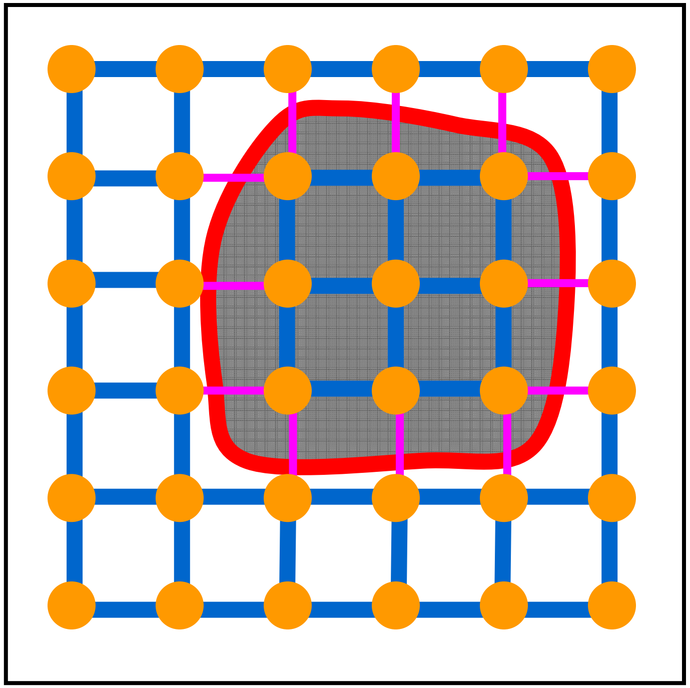
Minimize:

$$\Gamma(S) = \frac{E(S)}{N(S)}$$



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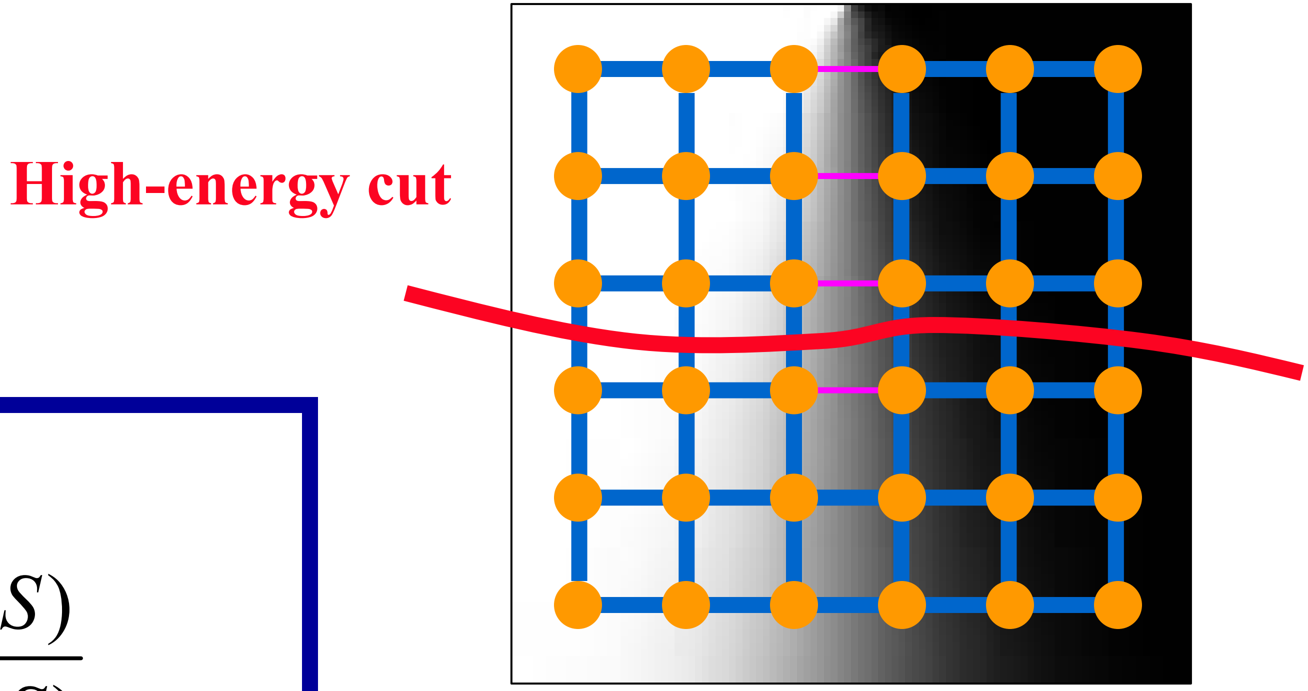
Normalized-Cut Measure



Minimize:

$$\Gamma(S) = \frac{E(S)}{N(S)}$$

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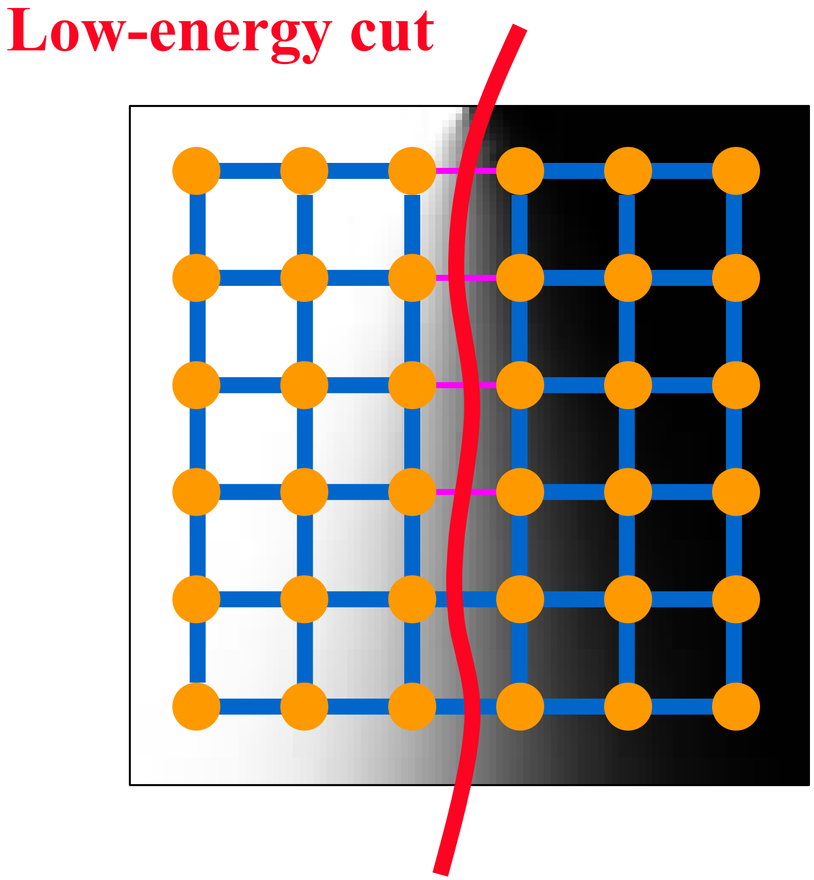
Normalized-Cut Measure



Minimize:

$$\Gamma(S) = \frac{E(S)}{N(S)}$$

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Matrix Formulation

Define matrix W by $\quad w_{ij} > 0 \quad w_{ii} = 0$

Define matrix L by $\quad l_{ij} = \begin{cases} \sum_{k,(k \neq i)} w_{ik} & i = j \\ -w_{ij} & i \neq j \end{cases}$

We minimize $\Gamma(u) = \dfrac{u^T L u}{\frac{1}{2} u^T W u}$

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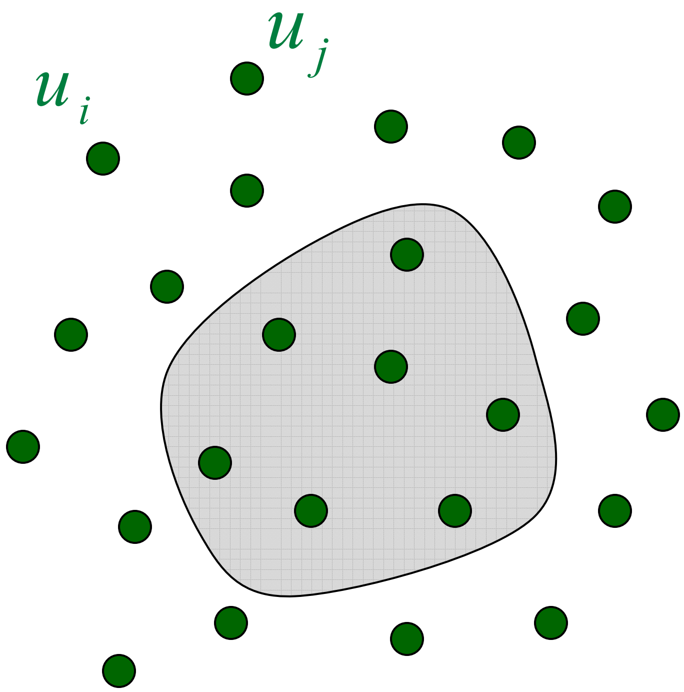
Coarsening the Minimization Problem



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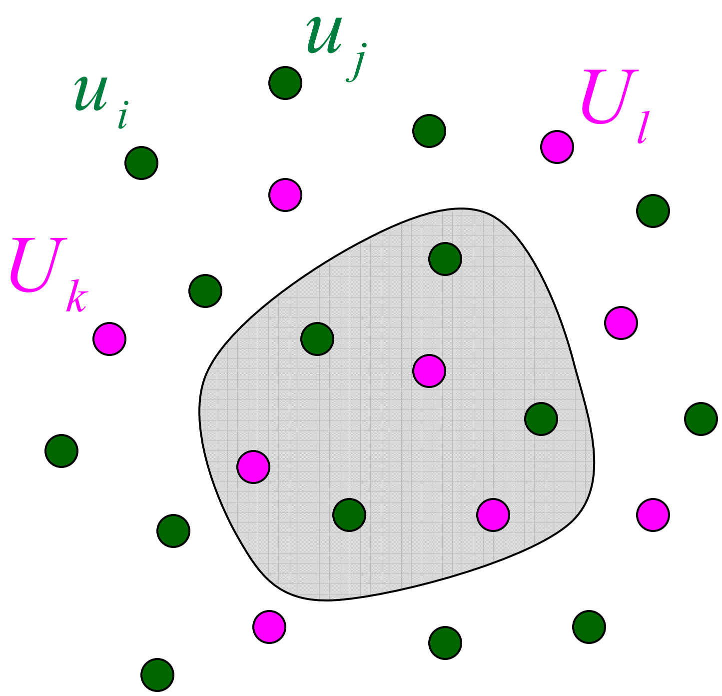
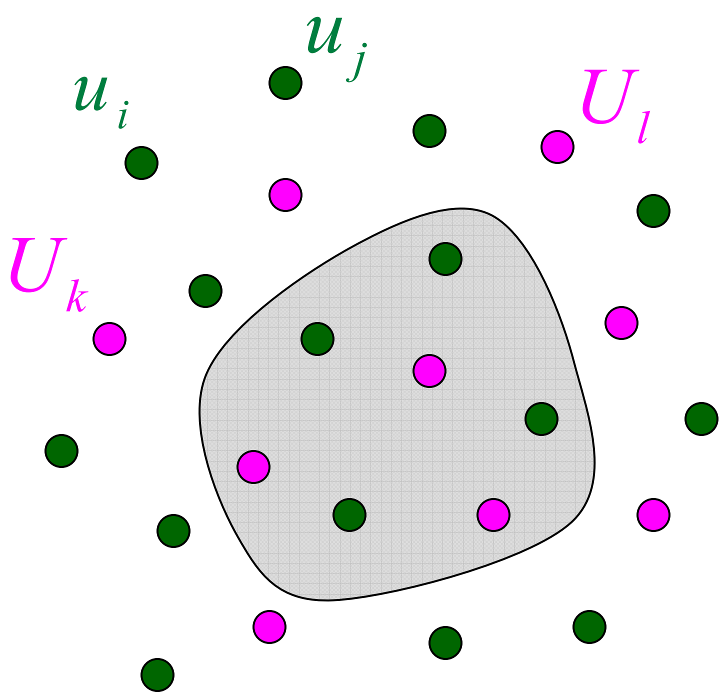
Coarsening – Choosing a Coarse Grid



Representative subset

$$U = (U_1, U_2, \ldots, U_N)$$

Coarsening –Interpolation Matrix



For a salient segment (globally minimizing solutions):

$$\begin{pmatrix} u_1 \\ u_2 \\ . \\ . \\ u_n \end{pmatrix} \simeq P \begin{pmatrix} U_1 \\ U_2 \\ . \\ U_N \end{pmatrix}$$

$P_{(n \times N)}$, **sparse interpolation matrix**

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Coarsening – Matrix Formulation

Given such an appropriate interpolation matrix P

$$\Gamma(u) = \frac{u^T L u}{\frac{1}{2} u^T W u} \approx \frac{U^T \left(P^T L P\right) U}{\frac{1}{2} U^T \left(P^T W P\right) U}$$

Existence of P from Algebraic Multigrid (AMG) for solving the equivalent Eigen-Value problem:

solve $Lu = \lambda D u$ for minimal $\lambda > 0$

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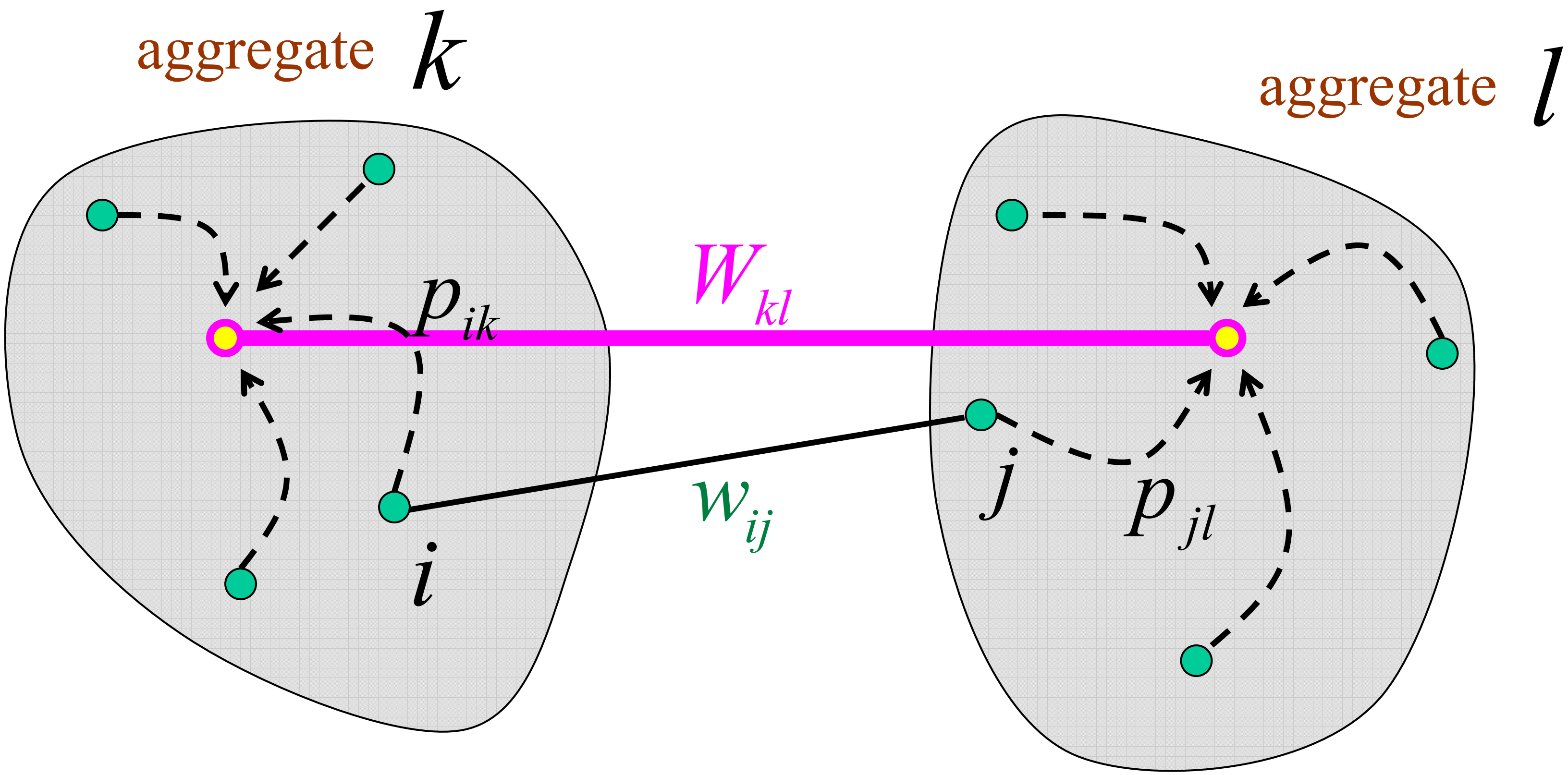
Weighted Aggregation



$$W_{kl} = \sum_{i \neq j} p_{ik} w_{ij} p_{jl}$$

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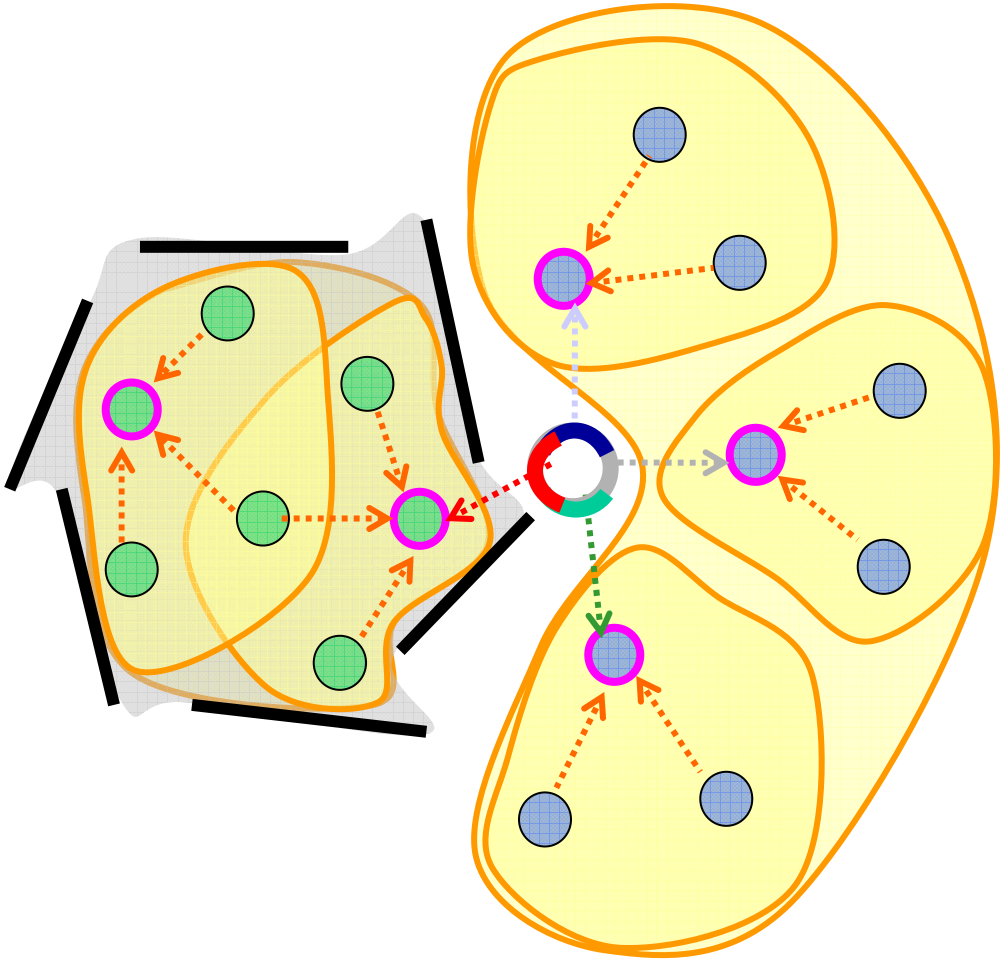
Importance of Soft Relations



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SWA

Detects the salient segments

Hierarchical structure

Linear in # of points
(a few dozen operations per point)

Coarse-Scale Measurements

- **Average intensities of aggregates**
- **Multiscale intensity-variances of aggregates**
- **Multiscale shape-moments of aggregates**
- **Boundary alignment between aggregates**

Coarse Measurements for Texture

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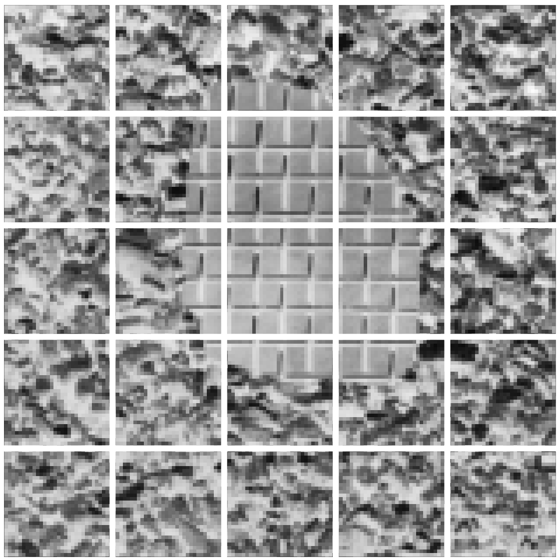
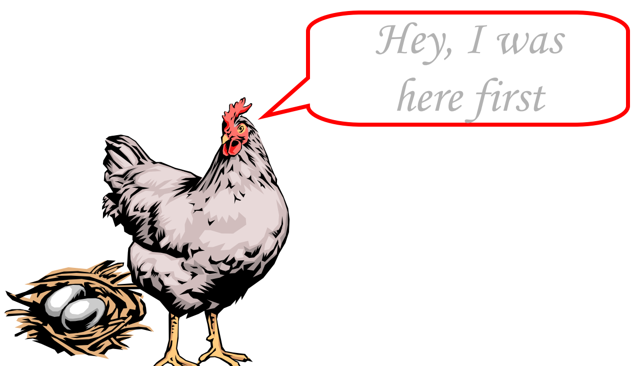
A Chicken and Egg Problem



Problem:
Coarse measurements mix neighboring statistics

Solution:
Support of measurements is determined as the segmentation process proceeds



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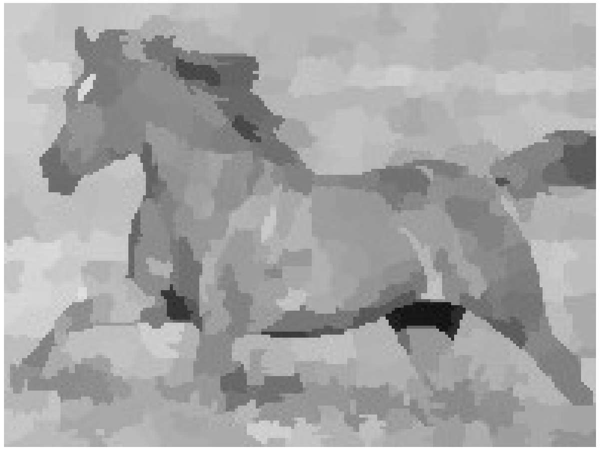
Adaptive vs. Rigid Measurements

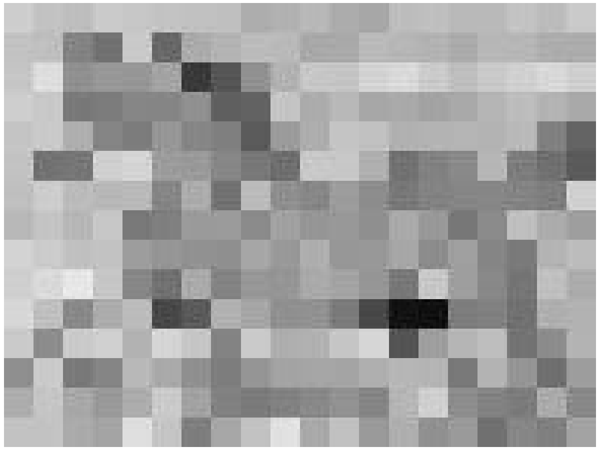
Original

Our algorithm - SWA



Averaging

Geometric



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Adaptive vs. Rigid Measurements

Original

Interpolation

Our algorithm - SWA

Geometric

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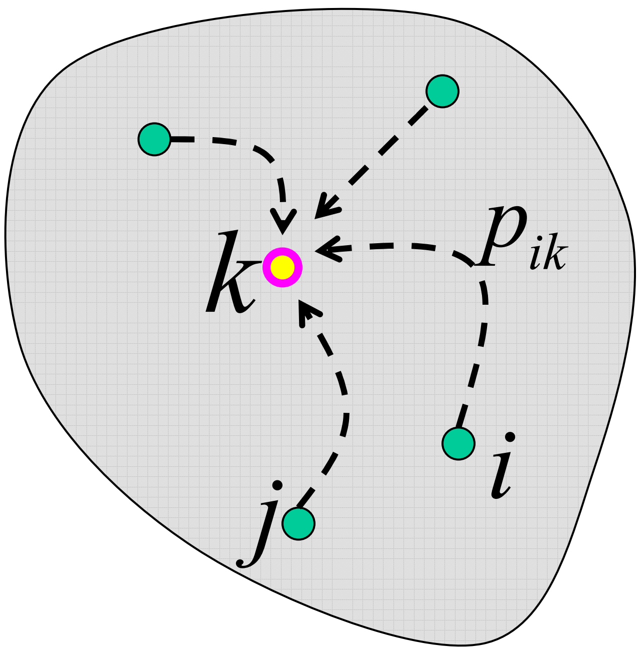
Recursive Measurements: Intensity

aggregate k



q_i intensity of pixel i

$$\bar{Q}_k = \frac{\sum_i p_{ik} q_i}{\sum_i p_{ik}}$$

average intensity of aggregate

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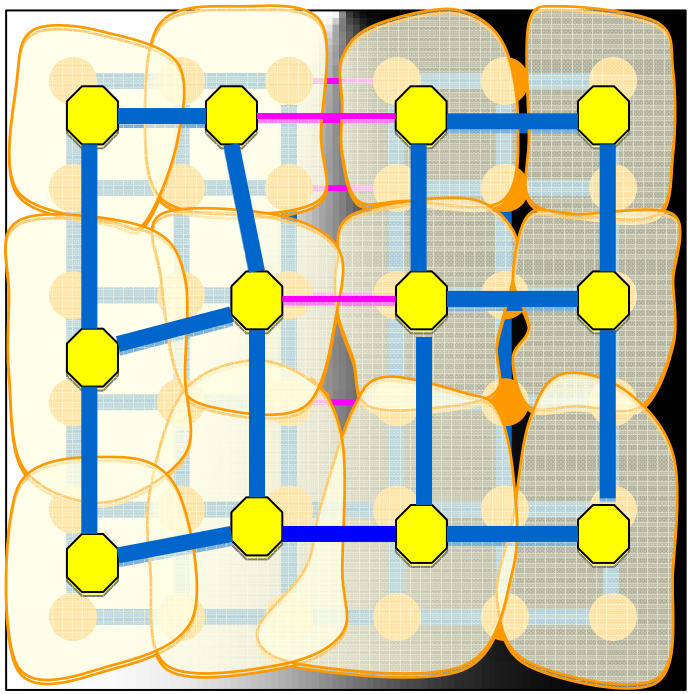
Use Averages to Modify the Graph



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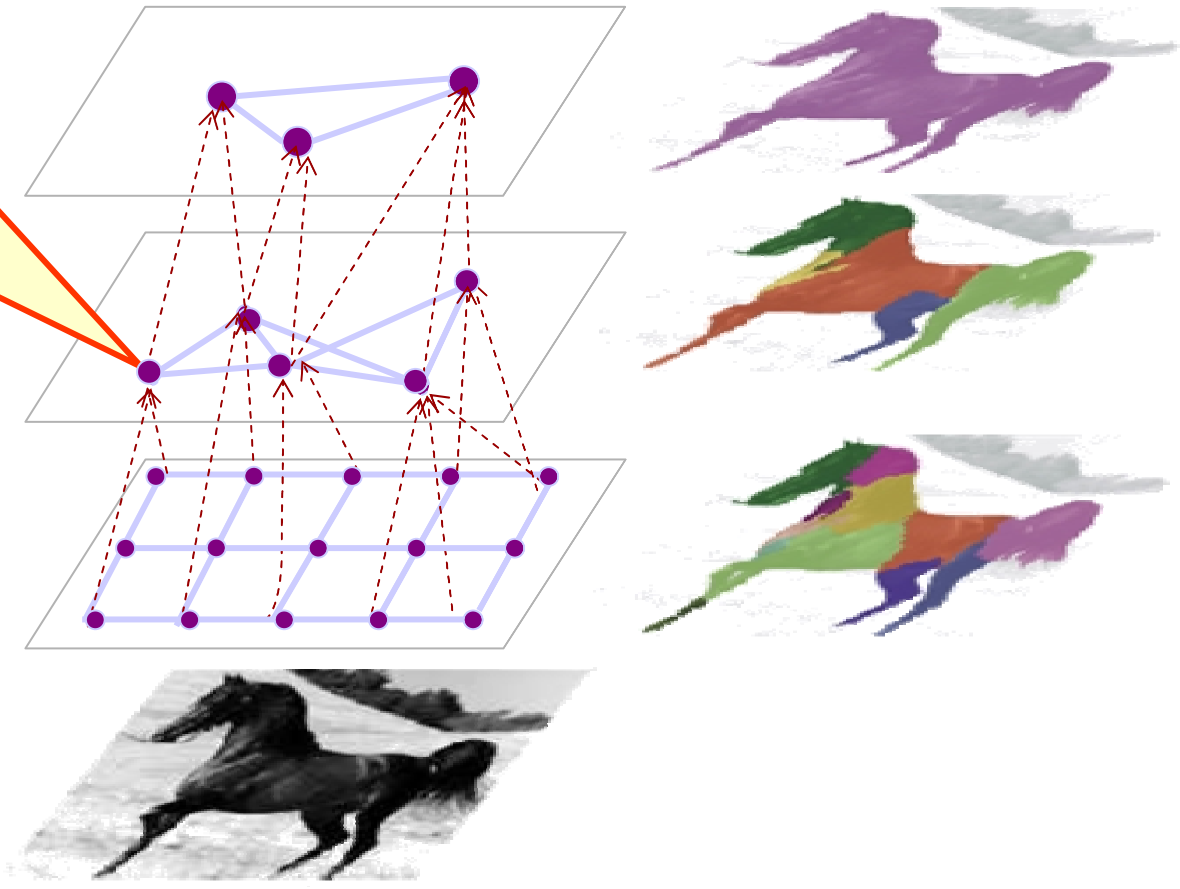
Hierarchy in **SWA**

- Average intensity
- Texture
- Shape



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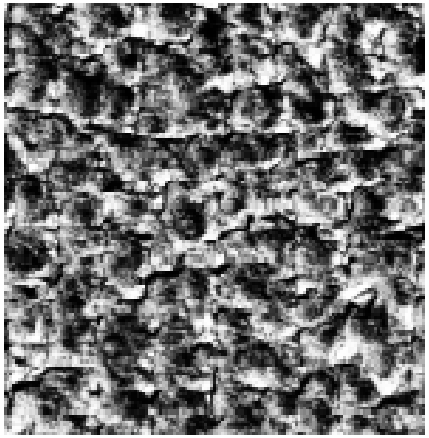
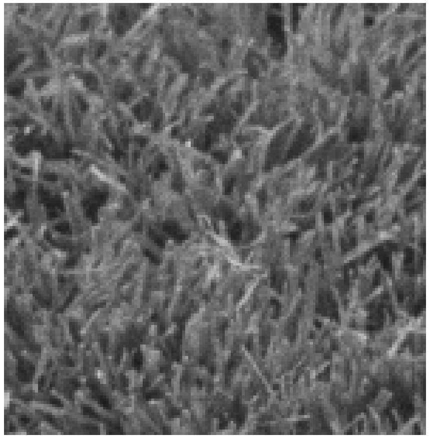
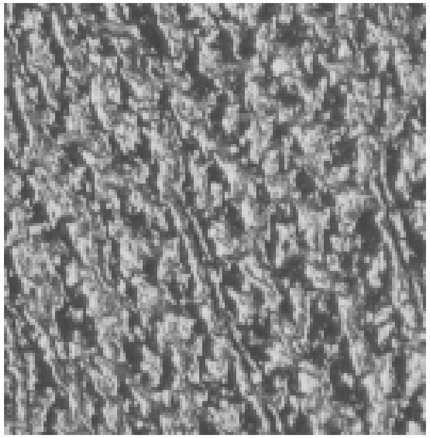
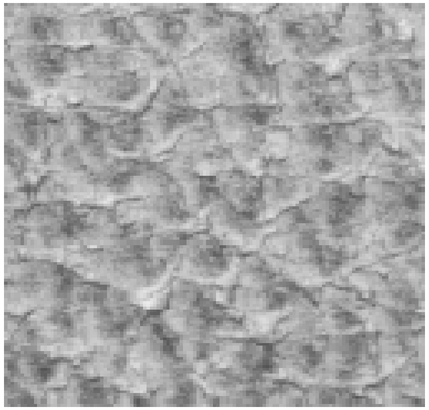
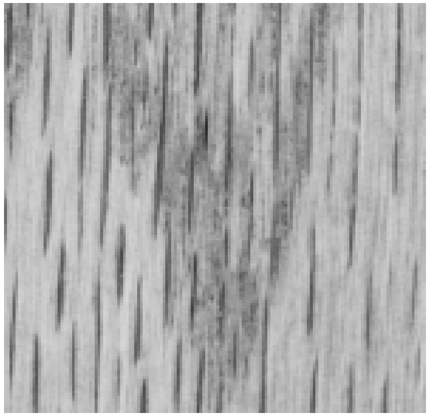
Texture Examples











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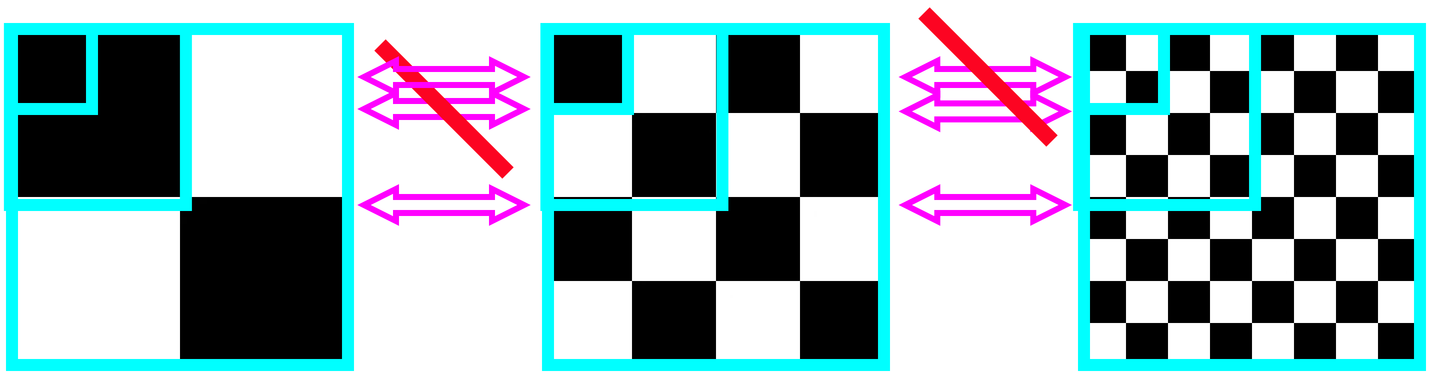
Isotropic Texture in SWA

Intensity Variance $V_k = \overline{\left(I^2\right)}_k - \left(\bar{I}_k\right)^2$



Isotropic Texture of aggregate –
average of variances in all scales

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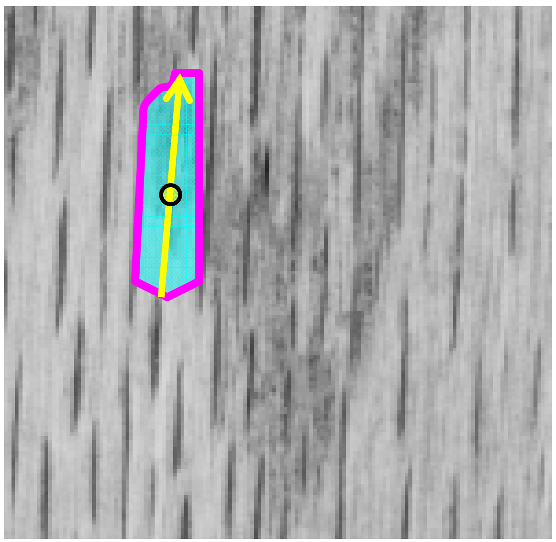
Oriented Texture in **SWA**

with Meirav Galun

Shape Moments $\langle x \rangle, \langle y \rangle, \langle xy \rangle, \langle x^2 \rangle, \langle y^2 \rangle$



- center of mass
- width
- length
- orientation

Oriented Texture of aggregate –
orientation, width and length in all scales

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Implementation

200×200 images on a Pentium III 1000MHz PC:

- **Our SWA algorithm (CVPR'00 + CVPR'01 + orientations Texture)**
  run-time: << 1 seconds.

400×400

run-time: 2-3 seconds.

Isotropic Texture

Our Algorithm (SWA)

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Isotropic Texture

Our Algorithm (SWA)

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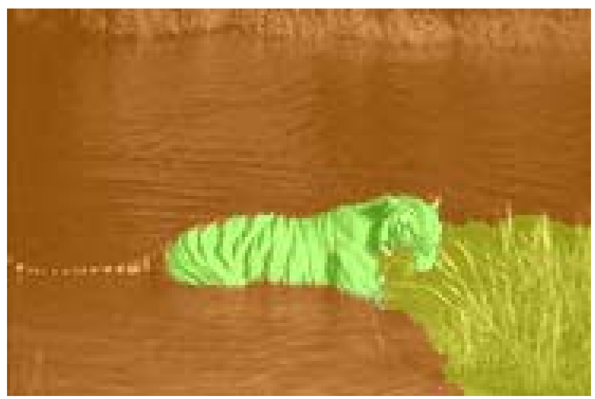
Isotropic Texture

Our Algorithm (SWA)



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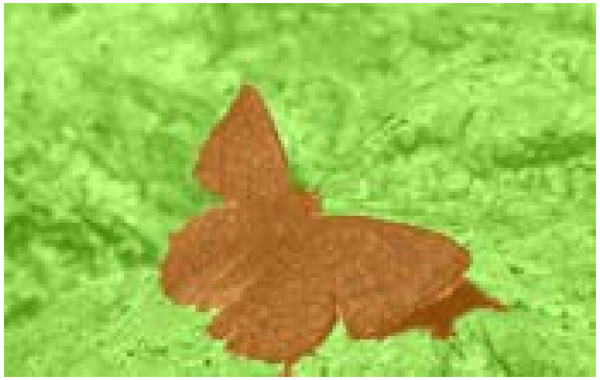
Isotropic Texture

Our Algorithm (SWA)



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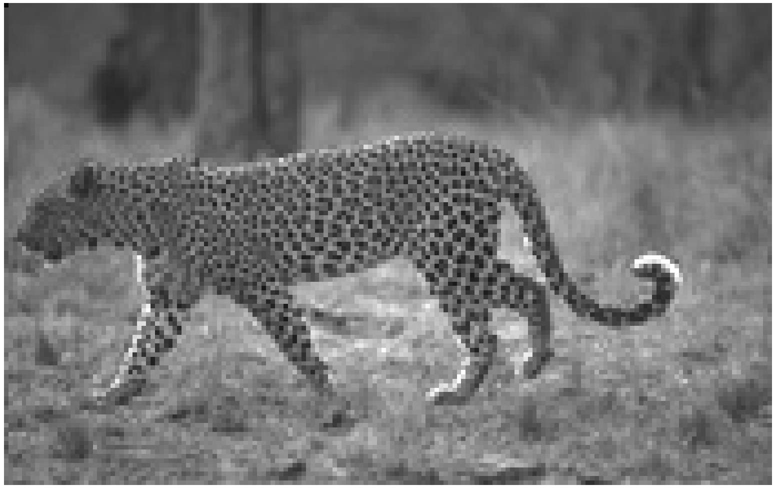
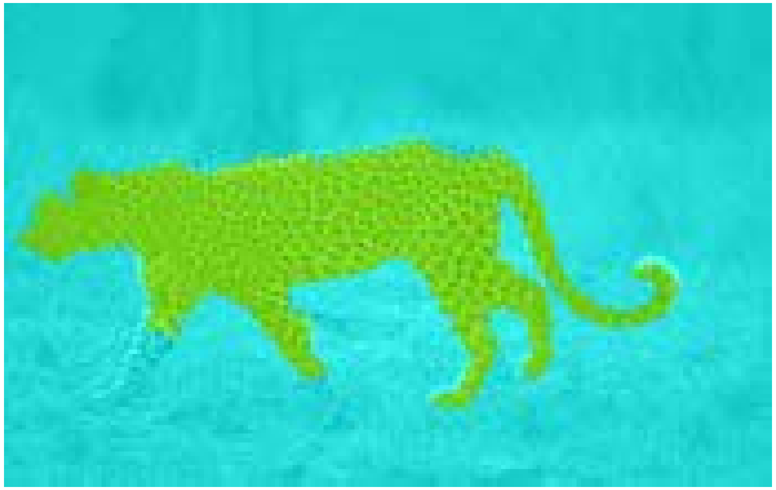
Isotropic Texture



Our Algorithm (SWA)



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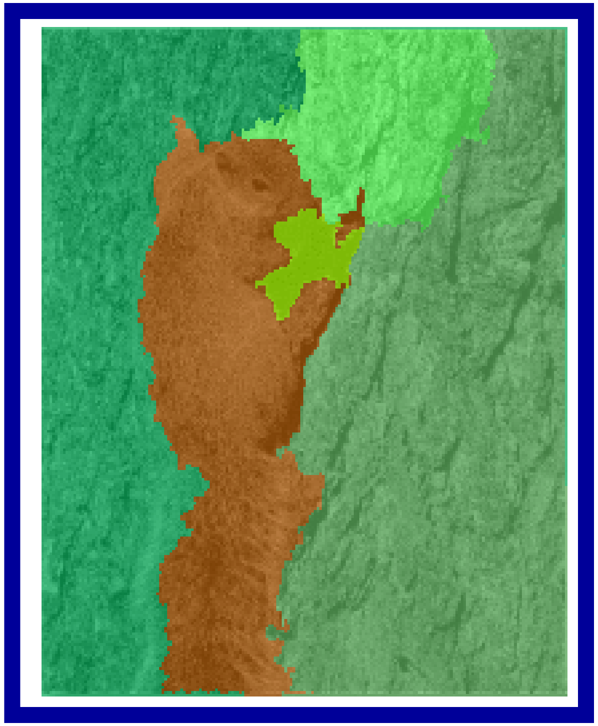
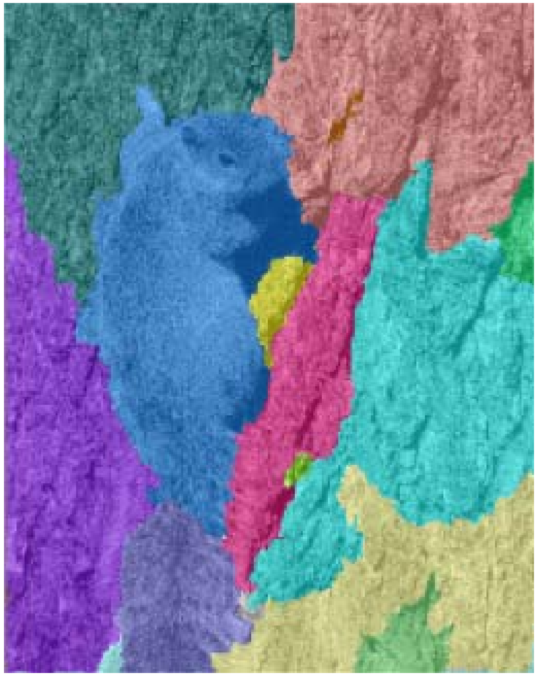


Our Algorithm (SWA)



Our previous

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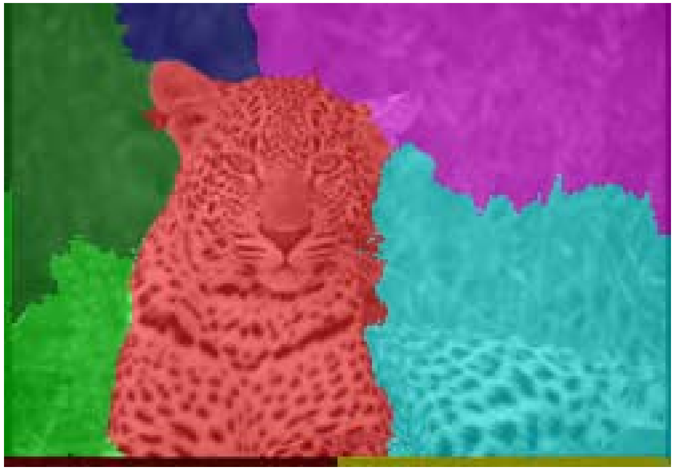
Our Algorithm (SWA)

Our previous



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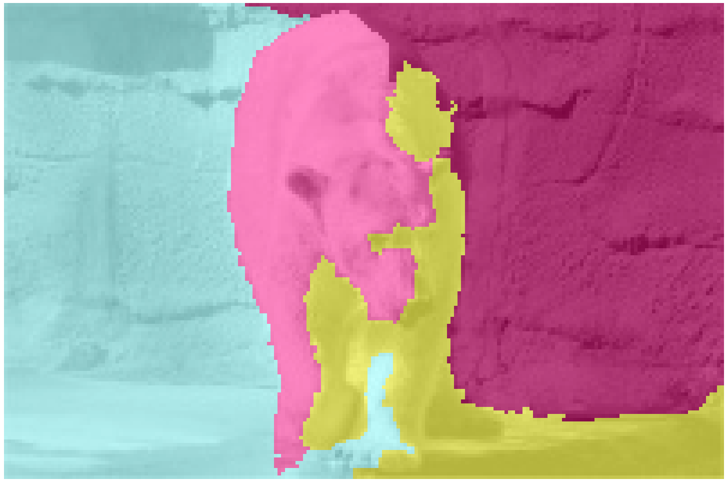
Our Algorithm (SWA)

Our previous



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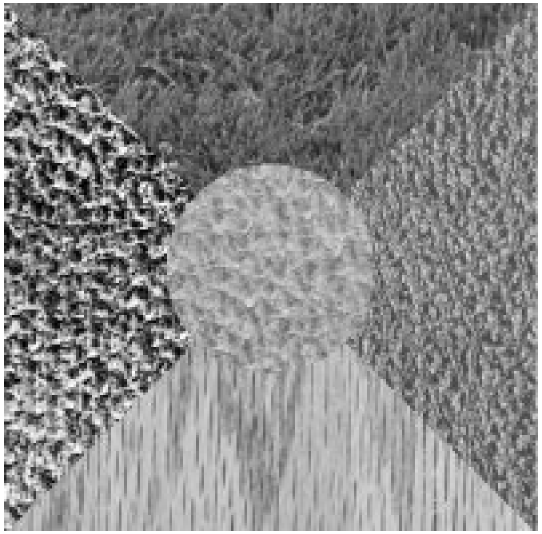
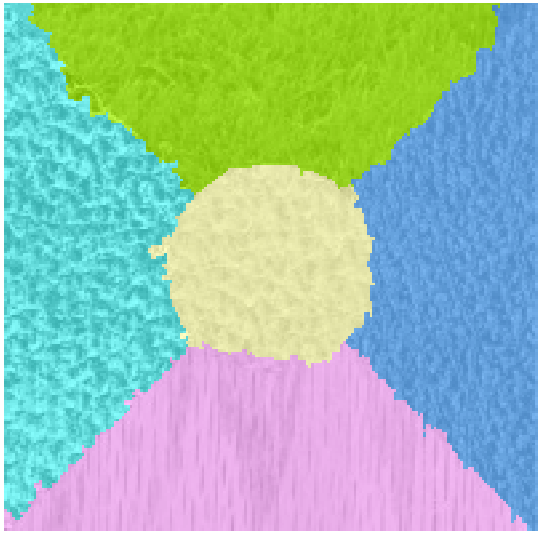


Our Algorithm (SWA)



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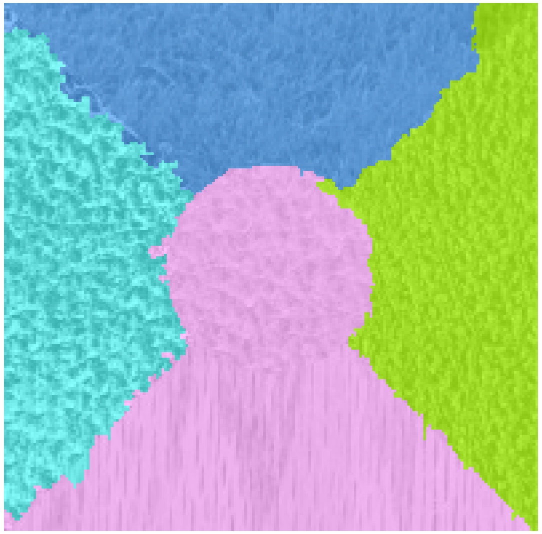

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