Problem 1 (10 points):
Let $A$ be the set of all English words. Let $R$ be a relation such that $aRb$ iff $a$ is at least as long as $b$.

Is $R$ a partial order?
- a linear order?
- an equivalence relation?

Problem 2 (10 points):
State whether each of the following is true or false:
1. $\phi \in \phi$
2. $\phi \subseteq \phi$
3. $\phi \in \{\phi\}$
4. $\{a, b\} \in P\{a, b, \{a, b\}\}$
5. $\{a, b\} \subseteq P\{a, b, \{a, b\}\}$

Problem 3 (25 points):
Show that in any group of people there are at least two persons that have the same number of acquaintances within the group. (Hint: Use the notion of one-to one correspondence.)

Problem 4 (20 points):
Construct a finite state automaton $M$ (i.e., the state diagram) with $\Sigma = \{a, b\}$ and $L(M) =$ the set of all strings that do NOT contain $aab$.

If $M$ is nondeterministic then construct $M'$ such that $M'$ is deterministic and $L(M) = L(M')$. 
Problem 5 (20 points):
Write a regular expression for the language accepted by the finite state machine, $M$ in Figure 1.

![Finite State Automaton](image)

Figure 1: Finite State Automaton M

Problem 6 (15 points):
Which one of the following are true?
1. $baa \in a^*b^*a^*b^*$
2. $b^*a^* \cap a^*b^* = a^* \cup b^*$
3. $a^*b^* \cap c^*d^* = \phi$