Regular Functions

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Regular Languages

- ❑ Natural

  Intuitive operational model of finite-state automata

- ❑ Robust

  Alternative characterizations and closure properties

- ❑ Analyzable

  Algorithms for emptiness, equivalence, minimization, learning …

- ❑ Applications

  Algorithmic verification, text processing …

What is the analog of regularity for defining functions?

Do we really need such a concept ?

FlashFill: Programming by Examples

Ref: Gulwani (POPL 2011)

Input	**Output**
Vechev, Martin	Martin Vechev
Martin Abadi	Martin Abadi
Rinard, Martin C.	Martin Rinard

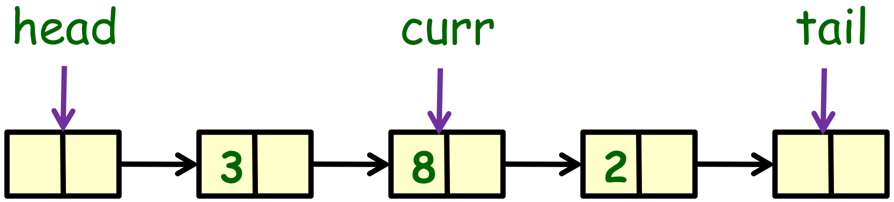
- ❑ Infers desired Excel macro program
- ❑ Iterative: user gives examples and corrections
- ❑ Already incorporated in Microsoft Excel

Learning regular languages	:	L* (Angluin'92)
Learning string transformation	:	??

Verification of List-processing Programs

```
function delete
  input ref curr;
  input data v;
  output ref result;
  output bool flag := 0;
  local ref prev;

  while (curr != nil) & (curr.data = v) {
      curr := curr.next;
      flag := 1;
      }
  result := curr;
  prev:= curr;
  if (curr != nil) then {
     curr := curr.next;
     prev.next := nil;
     while (curr != nil) {
         if (curr.data = v) then {
             curr := curr.next;
             flag := 1;
             }
         else {
             prev.next := curr;
             prev := curr;
             curr := curr.next;
             prev.next := nil;
             }
  }
```



Typically a simple function D* → D*

Insert

Delete

Reverse …

But finite-state verification algorithms not applicable, only lots of undecidability results !

Document Transformation

Task: Shift titles one entry up

```
@inproceedings{AC11,
   author = {Alur and Cerny},
   conference = {POPL 2011}
}

@inproceedings{AFR14,
   title = {Streaming transducers},
   conference = {LICS 2014},
   author = {Alur and Freilich and Raghothaman}
}

@inproceedings{ADR15,
   author = {Alur and D'Antoni and Raghothman},
   title = {Regular combinators},
   conference = {POPL 2015}
}
```

Should we use Perl ? sed ?
But these are Turing-complete languages with no "analysis" tools

Complexity Classification of Languages

|--- Recursive

|--- NP
|--- P

|--- Linear-time
|--- Regular

What if we consider functions?
From strings to strings

No essential change for
Recursive, NP, P, linear-time…

Natural starting point for regular functions:
Variation of classical finite-state automata

Finite-State Sequential Transducers

- Deterministic finite-state control + transitions labeled by (input symbol / string of output symbols)

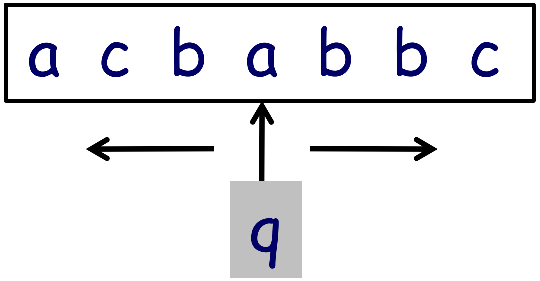
$$q \xrightarrow{a/010} q'$$

- Examples:
  - Delete all a symbols
  - Duplicate each symbol
  - Insert 0 after first b

- Theoretically not that different from classical automata, and have found applications in speech/language processing

Expressive enough ? What about reverse ?

Deterministic Two-way Transducers



- ☐ Unlike acceptors, two-way transducers more expressive than one-way model (Aho, Ullman 1969)
  - Reverse
  - Duplicate entire string (map w to w.w)
  - Delete a symbols if string ends with b (regular look-ahead)

Theory of Two-way Finite-state Transducers

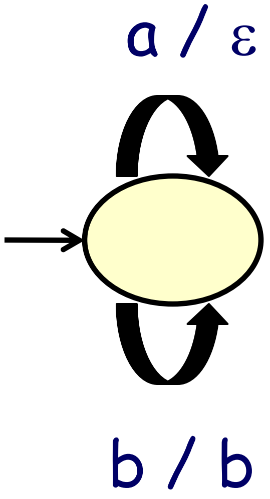
- Closed under sequential composition (Chytil, Jakl, 1977)
- Checking functional equivalence is decidable (Gurari 1980)
- Equivalent to MSO (monadic second-order logic) definable graph transductions (Engelfriet, Hoogeboom, 2001)
- Challenging theoretical results
  - Not like finite automata (e.g. Image of a regular language need not be regular !)
  - Complex constructions
  - No known applications

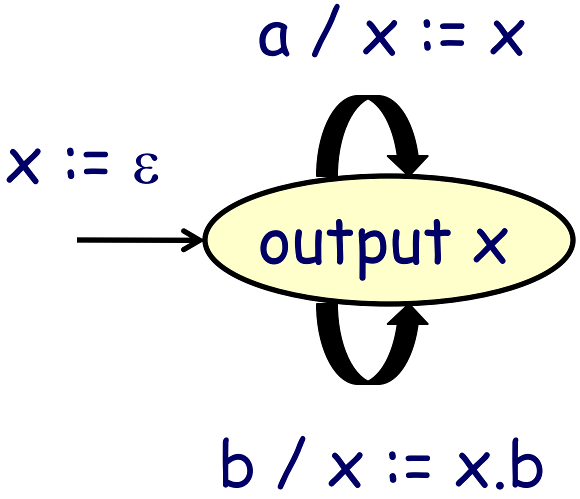
Talk Outline

- ➲ Machine model: Streaming String Transducers
- ☐ DReX: Declarative language for string transformations
- ☐ Regular Functions: Beyond strings to strings

Example Transformation 1: Delete

$Del_a(w)$ = String w with all a symbols removed



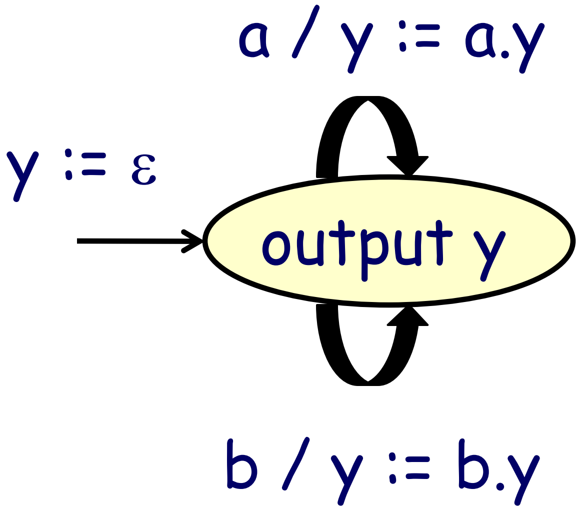


Traditional transducer

Finite-state control +
Explicit string variable to
compute output

Example Transformation 2: Reverse

Rev(w) = String w in reverse



a / y := a.y

y := ε

output y

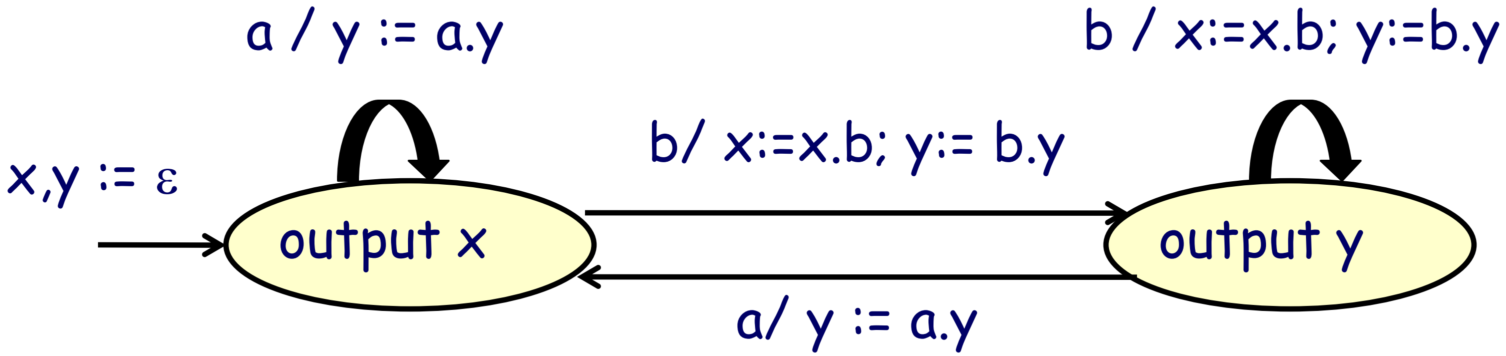
b / y := b.y

String variables updated at each step as in a program

Key restriction: No tests ! Write-only variables !

Example Transformation 3: Regular Choice

f(w)= If input ends with b, then Rev(w) else Del_a(w)

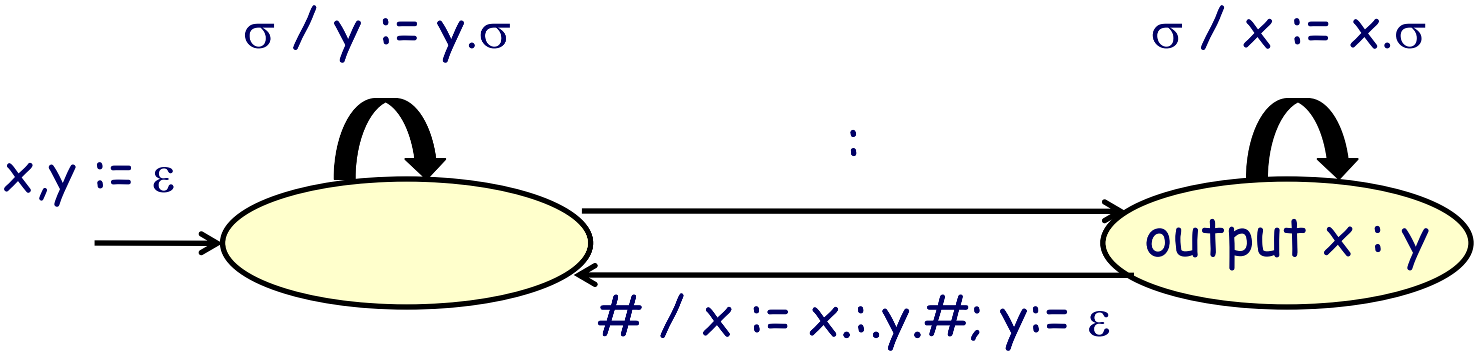


Multiple string variables used to compute alternative outputs

Model closed under “regular look-ahead”

Example Transformation 4: Swap

$f(u_1 : v_1 \# u_2 : v_2 \# \ldots) = v_1 : u_1 \# v_2 : u_2 \# \ldots$ u_i and v_i : $\{a,b\}^*$



σ / y := y.σ

σ / x := x.σ

x,y := ε

:

output x : y

/ x := x.:.y.#; y:= ε

Concatenation of string variables allowed (and needed)

Restriction: if x := x.y then y must be assigned a constant

Streaming String Transducer (SST)

1. Finite set Q of states
2. Input alphabet Σ
3. Output alphabet Γ
4. Initial state q_0
5. Finite set X of string variables
6. Partial output function $F : Q \rightarrow (\Gamma \cup X)^*$
7. State transition function $\delta : Q \times \Sigma \rightarrow Q$
8. Variable update function $\rho : Q \times \Sigma \times X \rightarrow (\Gamma \cup X)^*$

- ☐ Output function and variable update function required to be copyless: each variable x can be used at most once
- ☐ Configuration = (state q, valuation α from X to Γ^*)
- ☐ Semantics: Partial function from Σ^* to Γ^*

SST Properties

- ☐ At each step, one input symbol is processed, and at most a constant number of output symbols are newly created
- ☐ Output is bounded: Length of output = O(length of input)
- ☐ SST transduction can be computed in linear time
- ☐ Finite-state control: String variables not examined
- ☐ SST cannot implement merge

  $f(u_1u_2....u_k\#v_1v_2...v_k) = u_1v_1u_2v_2....u_kv_k$
- ☐ Multiple variables are essential

  For $f(w)=w^k$, k variables are necessary and sufficient

Decision Problem: Type Checking

Pre/Post condition assertion: { L } S { L' }

Given a regular language L of input strings (pre-condition), an SST S, and a regular language L' of output strings (post-condition), verify that for every w in L, S(w) is in L'

Thm: Type checking is solvable in polynomial-time

Key construction: Summarization

Decision Problem: Equivalence

Functional Equivalence;

Given SSTs S and S' over same input/output alphabets, check whether they define the same transductions.

Thm: Equivalence is solvable in PSPACE

(polynomial in states, but exponential in no. of string variables)

Open problem: Lower bound / Improved algorithm

Expressiveness

Thm: A string transduction is definable by an SST iff it is regular

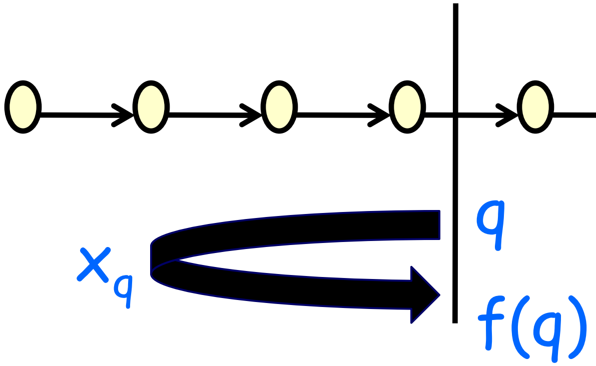
1. SST definable transduction is MSO definable
2. MSO definable transduction can be captured by a two-way transducer (Engelfriet/Hoogeboom 2001)
3. SST can simulate a two-way transducer

Evidence of robustness of class of regular transductions

Closure properties with effective constructions

1. Sequential composition: $f_1(f_2(w))$
2. Regular conditional choice: if w in L then $f_1(w)$ else $f_2(w)$

From Two-Way Transducers to SSTs



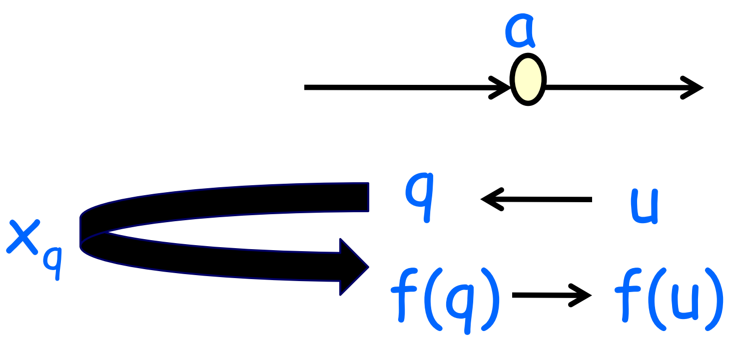
Two-way transducer A visits each position multiple times

What information should SST S store after reading a prefix?

For each state q of A, S maintains summary of computation of A started in state q moving left till return to same position

1. The state f(q) upon return
2. Variable x_q storing output emitted during this run

Challenge for Consistent Update



Map f: Q -> Q and variables x_q need to be consistently updated at each step

If transducer A moving left in state u on symbol a transitions to q, then updated f(u) and x_u depend on current f(q) and x_q

Problem: Two distinct states u and v may map to q

Then x_u and x_v use x_q, but assignments must be copyless !

Solution requires careful analysis of sharing (required value of each x_q maintained as a concatenation of multiple chunks)

Heap-manipulating Programs

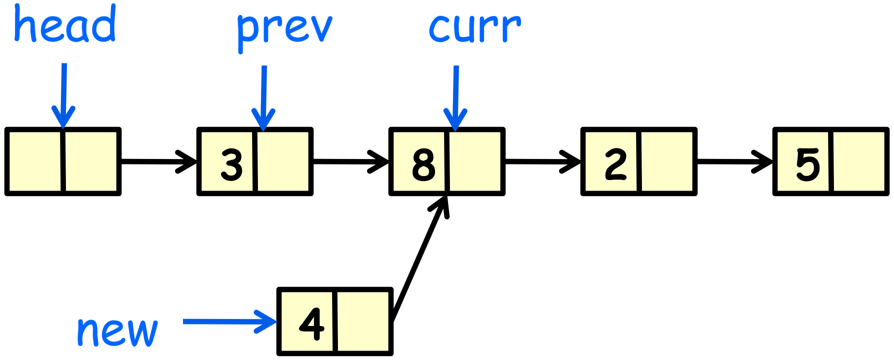
Sequential program +

Heap of cells containing data and next pointers +

Boolean variables +

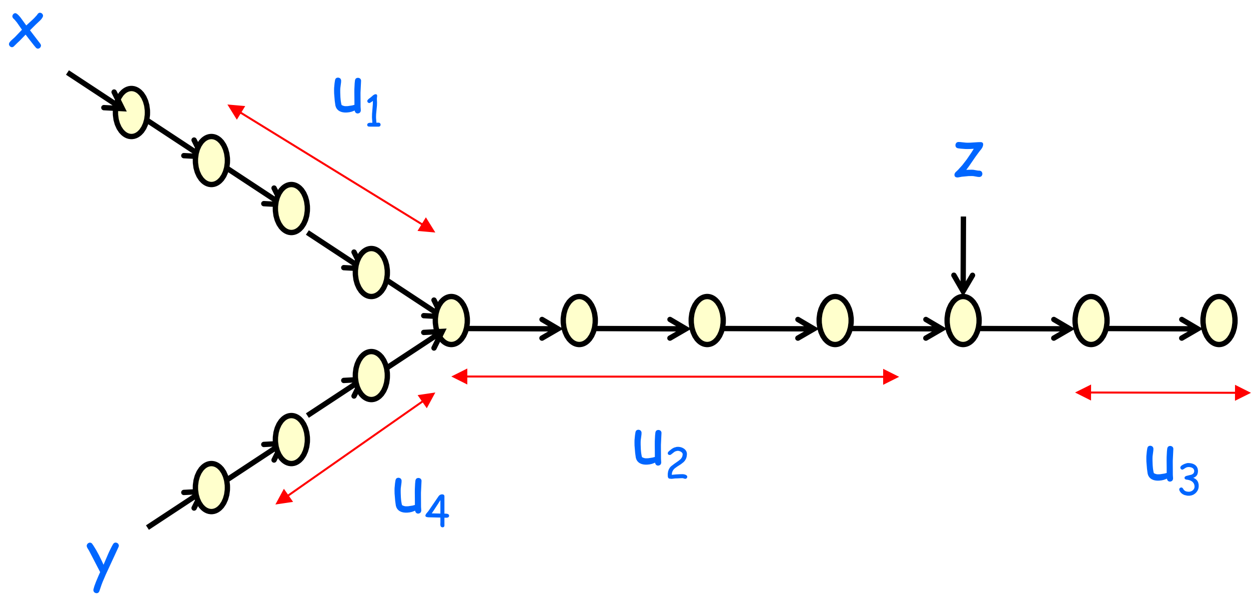
Pointer variables that reference heap cells

Program operations can add cells, change next pointers, and traverse the heap by following next pointers



How to restrict operations to capture exactly regular transductions

Representing Heaps in SST



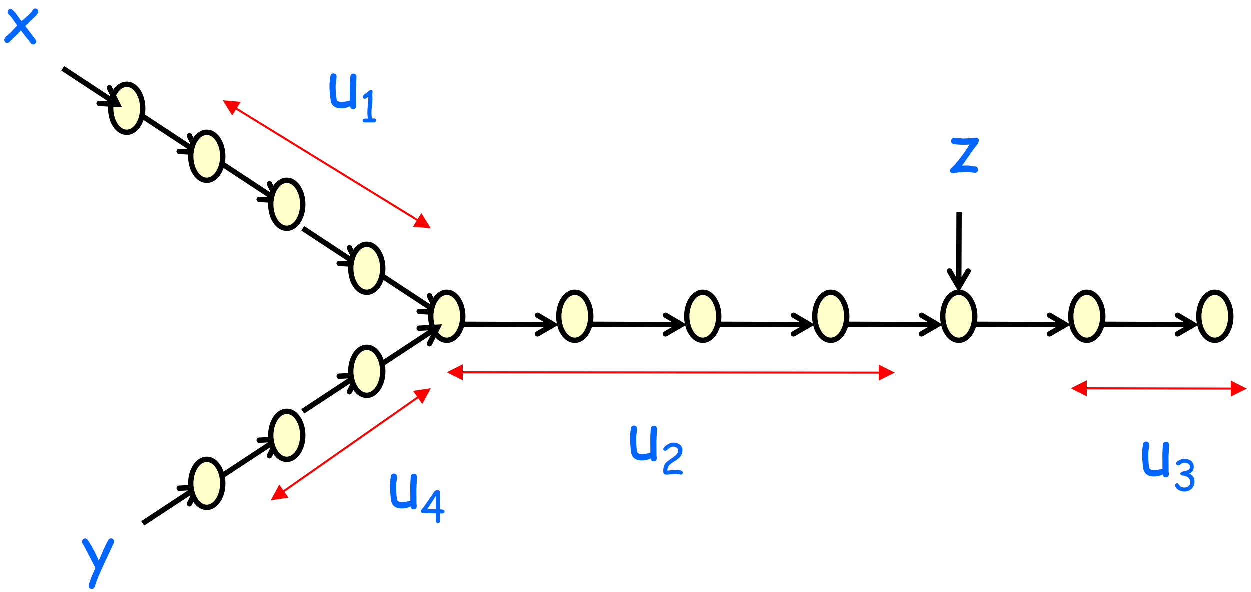
Shape (encoded in state of SST):

$x: u_1 u_2 z; \; y: u_4 u_2 z; \; z: u_3$

String variables: u_1, u_2, u_3, u_4

Shape + values of string vars enough to encode heap

Simulating Heap Updates

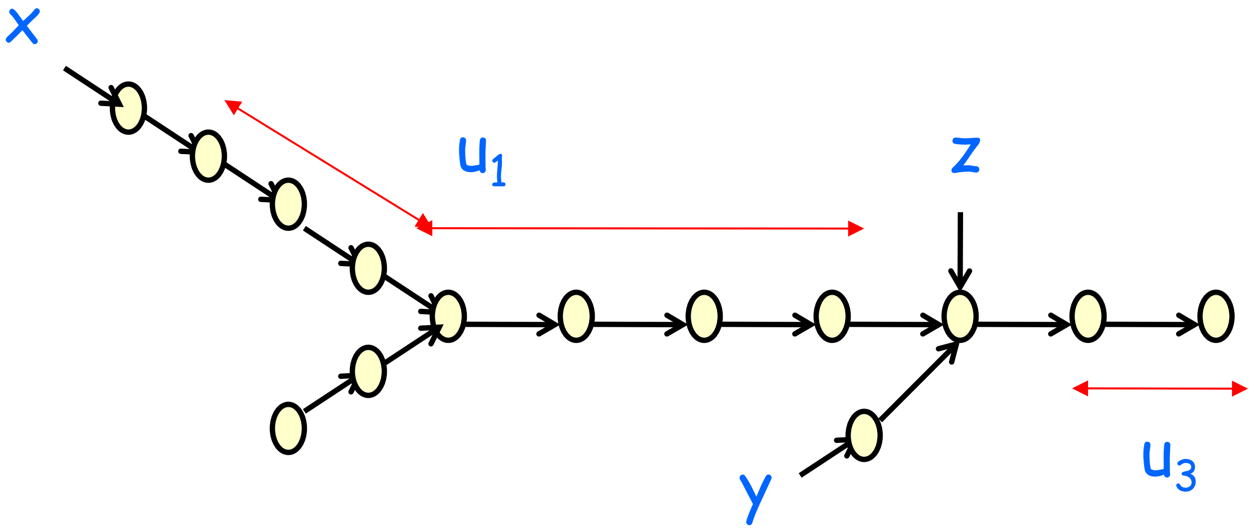


Consider program instruction

y.next := z

How to update shape and string variables in SST?

Simulating Heap Updates



New Shape: x: u_1 z ; y : z ; z : u_3

Variable update: $u_1 := u_1 u_2$

Special cells:

Cells referenced by pointer vars

Cells that 2 or more (reachable) next pointers point to

Contents between special cells kept in a single string var

Number of special cells = 2(# of pointer vars) - 1

Regular Heap Manipulating Programs

Update

x.next := y (changes heap shape destructively)

x := new (a) (adds new cell with data a and next nil)

Traversal

curr := curr.next (traversal of input list)

x := y.next (disallowed in general)

Theorem: Programs of above form can be analyzed by compiling into equivalent SSTs

Single pass traversal of input list possible

Pointers cannot be used as multiple read heads

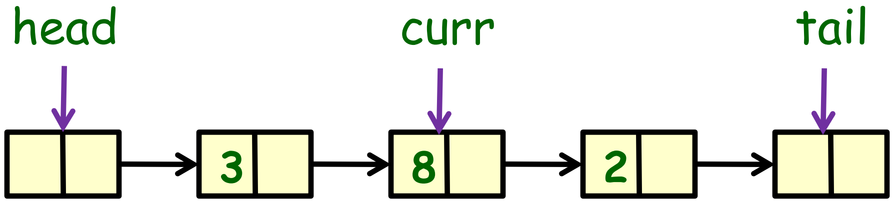
Manipulating Data

- ☐ Each string element consists of (tag t, data d)
  - Tags are from finite set
  - Data is from unbounded set D that supports = and < tests
  - Example of D: Names with lexicographic order
- ☐ SSTs and list-processing programs generalized to allow
  - Finite set of data variables
  - Tests using = and < between current value and data vars
  - Input and output values
- ☐ Checking equivalence remains decidable (in PSPACE) !
- ☐ Many common routines fall in this class
  - Check if list is sorted
  - Insert an element in a sorted list
  - Delete all elements that equal input value

Decidable Class of List-processing Programs

```
function delete
  input ref curr;
  input data v;
  output ref result;
  output bool flag := 0;
  local ref prev;

  while (curr != nil) & (curr.data = v) {
      curr := curr.next;
      flag := 1;
      }
  result := curr;
  prev:= curr;
  if (curr != nil) then {
     curr := curr.next;
     prev.next := nil;
     while (curr != nil) {
         if (curr.data = v) then {
             curr := curr.next;
             flag := 1;
             }
         else {
             prev.next := curr;
             prev := curr;
             curr := curr.next;
             prev.next := nil;
             }
  }
```



Decidable Analysis:

1. Assertion checks
2. Pre/post condition
3. Full functional correctness

Potential Application: String Sanitizers

- BEK: A domain specific language for writing string manipulating sanitizers on untrusted user data
- Analysis tool translates BEK program into (symbolic) transducer and checks properties such as
  - Is transduction idempotent: $f(f(w)) = f(w)$
  - Do two transductions commute: $f_1(f_2(w)) = f_2(f_1(w))$
- Recent success in analyzing IE XSS filters and other web apps
- Example sanitizer that BEK cannot capture (but SST can):

  Rewrite input w to suffix following the last occurrence of "dot"

Fast and precise sanitizer analysis with BEK.
Hooimeijer et al. USENIX Security 2011

Talk Outline

- ✓ Machine model: Streaming String Transducers
- ➲ DReX: Declarative language for string transformations
- ❑ Regular Functions: Beyond strings to strings

Search for Regular Combinators

- Regular Expressions
  - Basic operations: ε, a, Union, Concatenation, Kleene-*
  - Additional constructs (e.g. Intersection) : Trade-off between ease of writing constraints and complexity of evaluation

- What are the basic ways of combining functions?
  - Goal: Calculus of regular functions

- Partial function from Σ* to Γ*
  - Dom(f): Set of strings w for which f(w) is defined
  - In our calculus, Dom(f) will always be a regular language

Base Functions

- For a in Σ and γ in Γ^*, **a / γ**
  - If input w equals a then output γ, else undefined

- For γ in Γ^*, ε / γ
  - If input w equals ε then output γ else undefined

Choice

- **f else g**
  - Given input w, if w in Dom(f), then return f(w) else return g(w)
- Analog of union in regular expressions
  - Asymmetric (non-commutative) nature ensures that the result (f else g)(w) is uniquely defined
- Examples:
  - Id1 = (a / a) else (b / b)
  - Del_a1 = (a / ε) else Id1

Concatenation and Iteration

- **split (f, g)**
  - Given input string w, if there exist unique u and v such that w=u.v and u in Dom(f) and v in Dom(g) then return f(u).g(v)
  - Similar to "unambiguous" concatenation
- **iterate (f)**
  - Given input string w, if there is unique k and unique strings $u_1,\ldots u_k$ such that $w = u_1.u_2\ldots u_k$ and each u_i in Dom(f) then return $f(u_1)\ldots f(u_k)$
- **left-split (f, g)**
  - Similar to split, but return g(v).f(u)
- **left-iterate (f)**
  - Similar to iterate, but return $f(u_k)\ldots f(u_1)$

Examples

- Id1 = (a / a) else (b / b)
- Del_a1 = (a / ε) else Id1

- Id = iterate (Id1) : maps w to itself

- Del_a = iterate (Del_a1) : Delete all a symbols

- Rev = left-iterate (Id1) : reverses the input

- If w ends with b then delete a's else reverse

  split (Del_a, b / b) else Rev

- Map u#v to v.u

  left-split (split (Id, # / ε), Id)

Function Combination

- **combine (f, g)**
  - If w in both Dom(f) and Dom(g), then return f(w).g(w)

- combine(Id, Id) maps an input string w to w.w

- Needed for expressive completeness

- Reminiscent of Intersection for languages

Document Transformation Example

Task: Shift titles one entry up

```
@inproceedings{AC11,
   author = {Alur and Cerny},
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}

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   title = {Streaming transducers},
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}

@inproceedings{ADR15,
   author = {Alur and D'Antoni and Raghothman},
   title = {Regular combinators},
   conference = {POPL 2015}
}
```
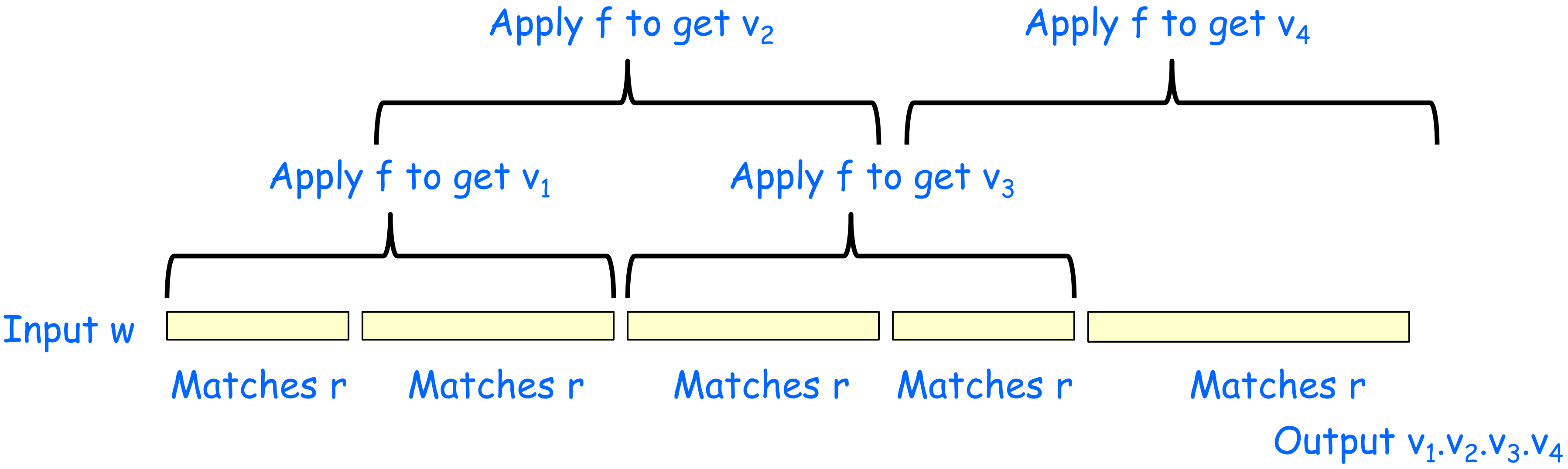

Does not seem expressible with combinators discussed so far…
Cannot compute this by splitting document in chunks, transforming them separately, and combining the results

Chained Iteration

chain (f, r) : Given input string w, if there is unique k and unique strings $u_1,\ldots u_k$ such that $w = u_1.u_2\ldots u_k$ and each u_i in Dom(r) then return $f(u_1u_2).f(u_2u_3)\ldots f(u_{k-1}u_k)$



Thm: A partial function $f : \Sigma^* \to \Gamma^*$ is regular iff it can be constructed using base functions, choice, split, left-split, combine, chain, and left-chain.

Towards a Prototype Language

- Goal: Design a DSL for regular string transformations

- Allow "symbolic" alphabet
  - Symbols range over a "sort"
  - Base function: $\varphi(x) / \gamma$
  - Set of allowed predicates form a Boolean algebra
  - Inspired by Symbolic Automata of Veanes et al

- Given a program P and input w, evaluation of P(w) should be fast!
  - Natural algorithm is based on dynamic programming: $O(|w|^3)$

Consistency Rules

- In **f else g**, Dom(f) and Dom(g) should be disjoint
- In **combine(f,g)**, Dom(f) and Dom(g) should be identical
- In **split(f,g)**, for every string w, there exists at most one way to split w = u.v such that u in Dom(f) and v in Dom(g)
- Similar rules for left-split, iterate, chain, and so on

DReX: Declarative Regular Transformations

- ❑ Syntax based on regular combinators + Type system to enforce consistency rules

- ❑ Thm: Restriction to consistent programs does not limit the expressiveness (DReX captures exactly regular functions)

- ❑ Consistency can be checked in poly-time in size of program

- ❑ For a consistent DReX program P, output P(w) can be computed in single-pass in time O(|w|) (and poly-time in |P|)
  - Intuition: To compute split(f,g)(w), whenever a prefix of w matches Dom(f), a new thread is started to evaluate g. Consistency is used to kill threads eagerly to limit the number of active threads

DReX Prototype Status

- Prototype implementation
  - Type checking
  - Linear-time evaluation

- Evaluation
  - How natural is it to write consistent DReX programs?
  - How does type checker / evaluator scale ?

- Ongoing work
  - Syntactic sugar with lots of pre-defined operations
  - Support for analysis (e.g. equivalence checking)
  - Integration in Python/Java ?

Talk Outline

✓ Machine model: Streaming String Transducers

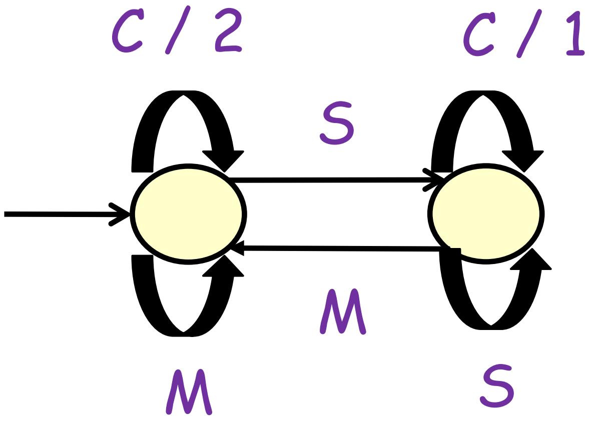
✓ DReX: Declarative language for string transformations

➲ Regular Functions: Beyond strings to strings

- Parameterized Definition of Regularity
- Additive Cost Register Automata
- Regular functions over a semi-ring

Mapping Strings to Numerical Costs

C: Buy Coffee
S: Fill out a survey
M: End-of-month

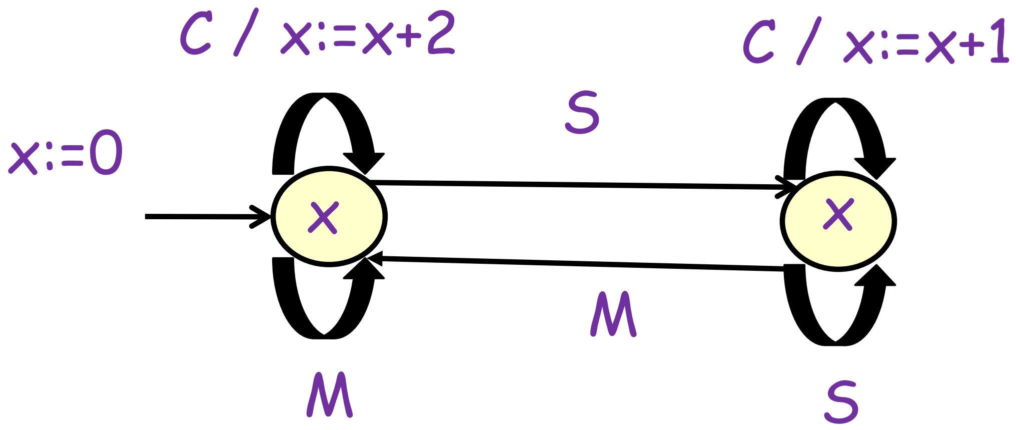


Maps a string over {C,S,M} to a cost value:

Cost of a coffee is 2, but reduces to 1 after filling out a survey until the end of the month

Can we generalize expressiveness using SST-style model?
Potential application: Quantitative analysis

Finite Automata with Cost Registers



Cost Register Automata:

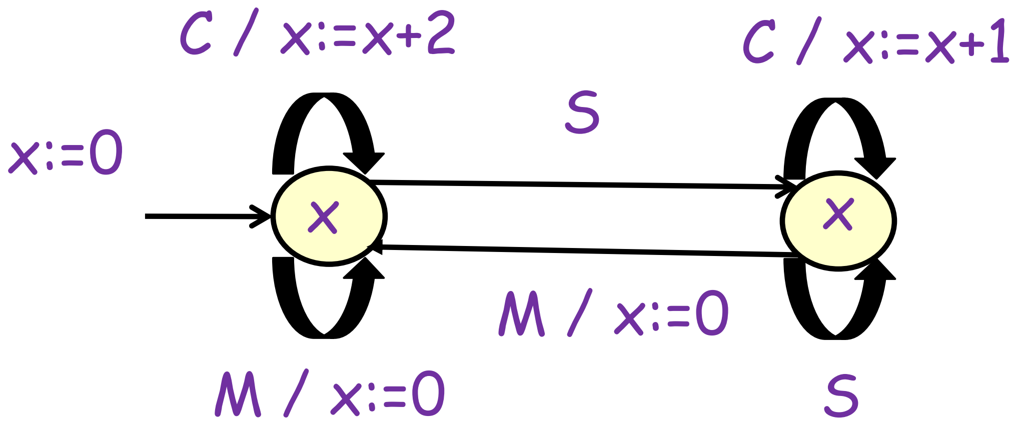
Finite control + Finite number of registers

Registers updated explicitly on transitions

Registers are write-only (no tests allowed)

Each (final) state associated with output register

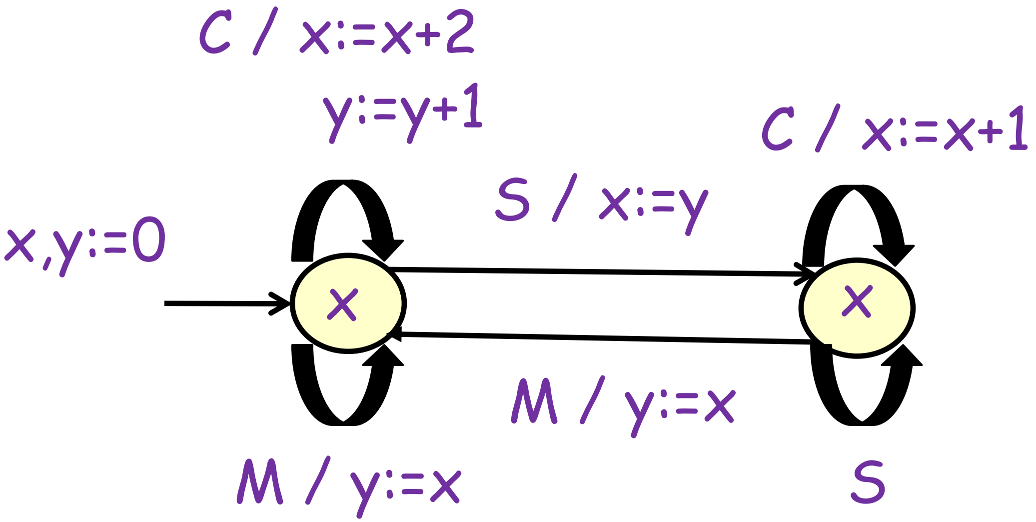
CRA Example



At any time, x = cost of coffees during the current month

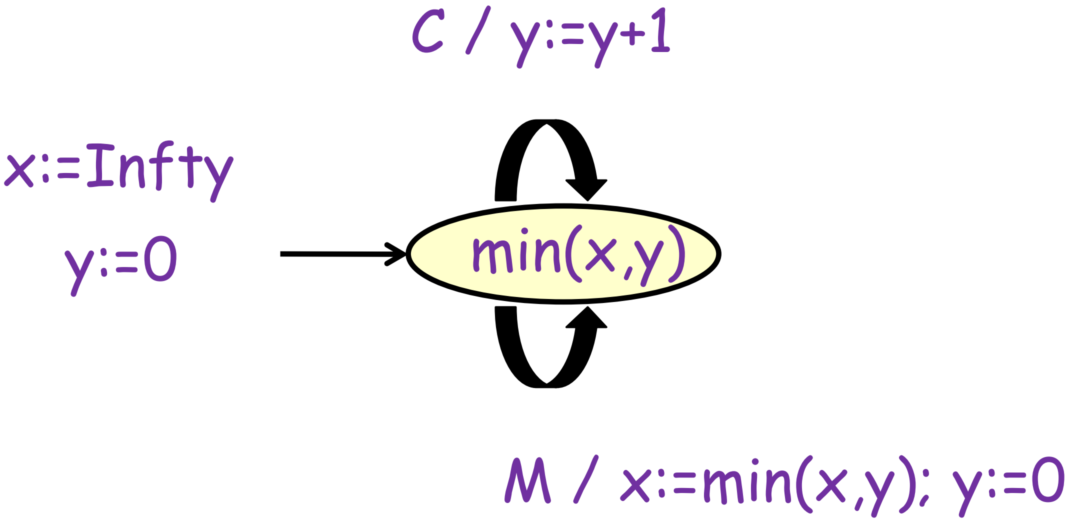
Cost register x reset to 0 at each end-of-month

CRA Example



Filling out a survey gives discount for all coffees during that month

CRA Example



C / y:=y+1

x:=Infty

y:=0

min(x,y)

M / x:=min(x,y); y:=0

Output = minimum number of coffees consumed during a month

Updates use two operations: increment and min

Can we define a general notion of regularity parameterized by operations on the set of costs ?

Cost Model

Cost Grammar G to define set of terms:

Inc: t := c | (t+c)

Plus: t := c | (t+t)

Min-Inc: t := c | (t+c) | min(t,t)

Inc-Scale: t := c | (t+c) | (t*d)

Interpretation [] for operations:

Set D of cost values

Mapping operators to functions over D

Example interpretations for the Plus grammar:

Set N of natural numbers with addition

Set Γ^* of strings with concatenation

Regular Function

Definition parameterized by the cost model *C*=(D,*G*,[])

A (partial) function f:Σ*->D is regular w.r.t. the cost model *C* if there exists a string-to-tree transformation *g* such that

(1) for all strings w, f(w)=[*g*(w)]

(2) *g* is a regular string-to-tree transformation

MSO-definable String-to-tree Transformations

- ☐ MSO over strings

  Φ := a(x) | X(x) | x=y+1 | ~ Φ | Φ & Φ | Exists x. Φ | Exists X. Φ

- ☐ MSO-transduction from strings to trees:

  1. Number k of copies

     For each position x in input, output-tree has nodes x_1, …x_k

  2. For each symbol a and copy c, MSO-formula $\Phi_{a,c}(x)$

     Output-node x_c is labeled with a if $\Phi_{a,c}(x)$ holds for unique a

  3. For copies c and d, MSO-formula $\Phi_{c,d}(x,y)$

     Output-tree has edge from node x_c to node x_d if $\Phi_{c,d}(x,y)$ holds

Example Regular Function

Cost grammar Min-Inc: t := c | (t+c) | min(t,t)

Interpretation: Natural numbers with usual meaning of + and min

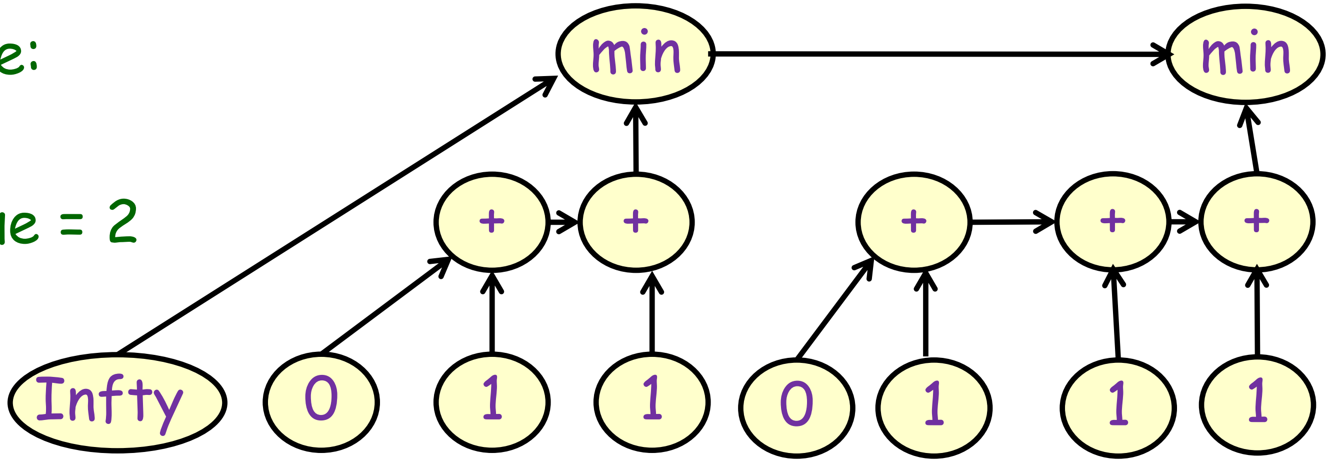
Σ={C,M}

f(w) = Minimum number of C symbols between successive M's

Input w= C C M C C C M

Tree:

Value = 2



Regular String-to-tree Transformations

- Definition based on MSO (Monadic Second Order Logic) – definable graph-to-graph transformations (Courcelle)
- Studied in context of syntax-directed program transformations, attribute grammars, and XML transformations
- Operational model: Macro Tree Transducers (Engelfriet et al)
- Recent proposal: Streaming Tree Transducers (ICALP 2012)

Properties of Regular Functions

Known properties of regular string-to-tree transformations imply:

- ☐ If f and g are regular w.r.t. a cost model C, and L is a regular language, then "if L then f else g" is regular w.r.t. C
- ☐ Reversal: define Rev(f)(w) = f(reverse(w)).
  If f is regular w.r.t. a cost model C, then so is Rev(f)
- ☐ Costs grow linearly with the size of the input string:
  Term corresponding to a string w is O(|w|)

Regular Functions for Non-Commutative Monoid

- Cost model: Γ^* with binary function concatenation
- Interpretation for . is non-commutative, associative, identity ε
- Cost grammar G(.): $t := \sigma \mid (t \,.\, t)$ σ is a string
- Cost grammar G(.σ): $t := \sigma \mid (t \,.\, \sigma) \mid (\sigma \,.\, t)$
- Thm: Regular functions w.r.t G(.) is a strict superset of regular functions w.r.t. G(.σ)
- Classical model of Sequential Transducers captures only a subset of regular functions w.r.t. G(.σ)
- SSTs capture exactly regular functions w.r.t. G(.)

Regular Functions over Commutative Monoid

Cost model: D with binary function +

Interpretation for + is commutative, associative, with identity 0

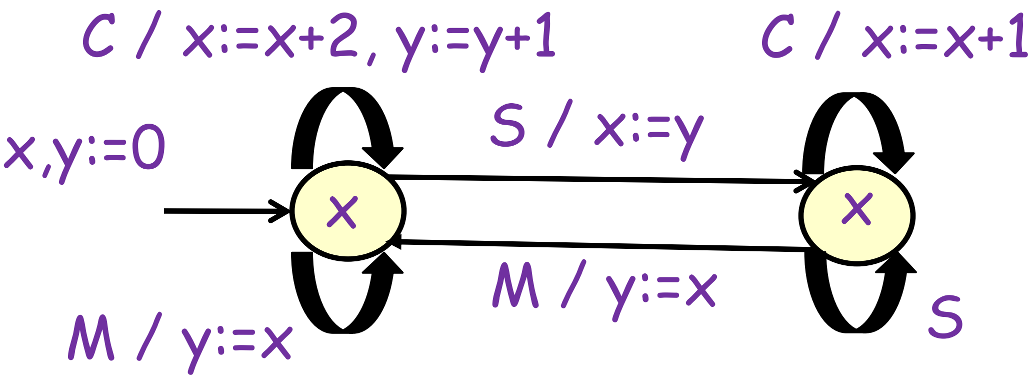
Cost grammar G(+): t := c | (t+t)

Cost grammar G(+c): t := c | (t+c)

Thm: Regularity w.r.t. G(+) coincides with regularity w.r.t. G(+c)

Proof intuition: Show that rewriting terms such as (2+3)+(1+5) to (((2+3)+1)+5) is a regular tree-to-tree transformation, and use closure properties of tree transducers

Additive Cost Register Automata



Additive Cost Register Automata:

DFA + Finite number of registers

Each register is initially 0

Registers updated using assignments x := y + c

Each final state labeled with output term x + c

Given commutative monoid (D,+,0), an ACRA defines a partial function from Σ^* to D

Regular Functions and ACRAs

- Thm: Given a commutative monoid (D,+,0), a function f:Σ*->D is definable using an ACRA iff it is regular w.r.t. grammar G(+).

- Establishes ACRA as an intuitive, deterministic operational model to define this class of regular functions

- Proof relies on the model of SSTT (Streaming string-to-tree transducers) that can define all regular string-to-tree transformations

Single-Valued Weighted Automata

- Weighted Automata:
  - Nondeterministic automata with edges labeled with costs
- Single-valued:
  - Each string has at most one accepting path
- Cost of a string:
  - Sum of costs of transitions along the accepting path
- Example: When you fill out a survey, each coffee during that month gets the discounted cost.
  - Locally nondeterministic, but globally single-valued
- Thm: ACRAs and single-valued weighted automata define the same class of functions

Decision Problems for ACRAs

- ❑ Min-Cost: Given an ACRA M, find min {M(w) | w in Σ^*}

  Solvable in Polynomial-time

  Shortest path in a graph with vertices (state, register)

- ❑ Equivalence: Do two ACRAs define the same function

  Solvable in Polynomial-time

  Based on propagation of linear equalities in program graphs

- ❑ Register Minimization: Given an ACRA M with k registers, is there an equivalent ACRA with < k registers?

  Algorithm polynomial in states, and exponential in k

Towards a Theory of Additive Regular Functions

- Goal: Machine-independent characterization of regularity
  - Similar to Myhill-Nerode theorem for regular languages
  - Registers should compute necessary auxiliary functions

- Example: $\Sigma = \{C, S\}$
  - $f(w)$= if w contains S then $|w|$ else $2|w|$
  - $f_1(C^i)=i$ and $f_2(C^i)=2i$ are necessary and sufficient

- Thm: Register complexity of a function is at least k iff there exist strings $\sigma_0, \ldots \sigma_m$, loop-strings $\tau_1, \ldots \tau_m$, and suffixes $w_1, \ldots w_m$, and k distinct vectors $\mathbf{c}_1, \ldots \mathbf{c}_k$ such that for all numbers $x_1, \ldots x_m$, $f(\sigma_0\ \tau_1^{x1}\ \sigma_1\ \tau_2^{x2} \ldots \sigma_m\ w_i) = \Sigma_j\ c_{ij}\ x_j + d_i$

Regular Functions over Semiring

- Cost Domain: Natural numbers + Infty
- Operation Min: Commutative monoid with identity Infty
- Operation +: Monoid with identity 0
- Rules:

  a + Infty = Infty + a = Infty
  a+min(b,c) = min (a+b, a+c); min(b,c)+a = min(b+a,c+a)

- Cost grammar MinInc: t := c | min(t,t) | (t+c)
- Goal: Understand class of regular functions w.r.t. MinInc

Weighted Automata

- ❑ Weighted Automata:

  Nondeterministic automata with edges labeled with costs

- ❑ Interpreted over the semiring cost model:

  cost of string w = min of costs of all accepting paths over w

  cost of a path = sum of costs of all edges in a path

- ❑ Widely studied (Handbook of Weighted Automata, Droste et al)

  Minimum cost problem solvable

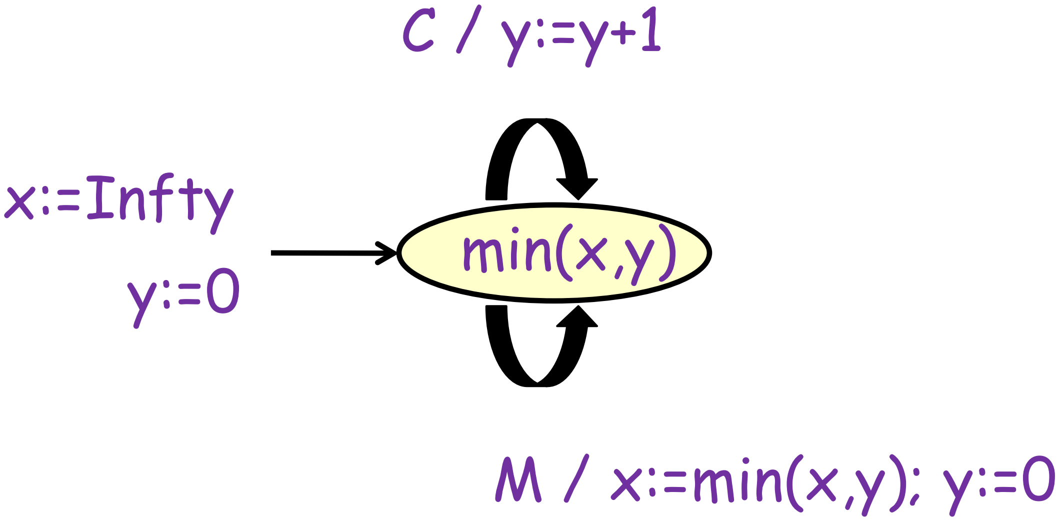
  Equivalence undecidable over (N, min, +)

  Not determinizable

  Natural model in many applications

  Recent interest in CAV/LICS community for quantitative analysis

CRA over Min-Inc Semiring



C / y:=y+1

x:=Infty
y:=0

min(x,y)

M / x:=min(x,y); y:=0

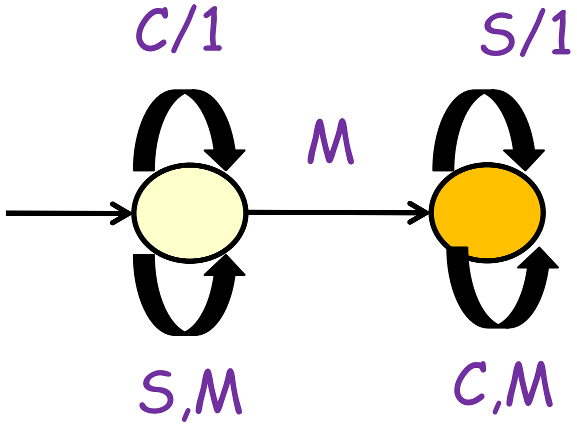
Output = Minimum number of coffees consumed during a month

CRA(min,+c) = Weighted Automata

- ☐ From WA to CRA(min,+c):
  - Generalizes subset construction for determinization
  - For every state q of WA, CRA maintains a register x_q
  - x_q = min of costs of all paths to q on input read so far
  - Update on a: $x_q := \min \{ x_p + c \mid p \xrightarrow{(a,c)} q$ is edge in WA$\}$

- ☐ From CRA(min,+c) to WA:
  - State of WA = (state q of CRA, register x)
  - min simulated by nondeterminism
  - To simulate p – (a, x:=min(y,z)) -> q in CRA,
    - add a-labeled edges from (p,y) and (p,z) to (q,x)
  - Distributivity of + over min critical

CRA(min,+c) > Min-Plus Regular Functions



Input w: w_1 M w_2 M … M w_n

Each w_i in {C,S}*

c_i = Number of C's in w_i

s_i = Number of S's in w_i

Cost(w) = $\min_j \{ c_1+\ldots+c_j+s_{j+1}+\ldots+s_n\}$

Thm: The class of regular functions w.r.t. Min-Inc semiring is a strict subset of weighted automata

Above function is not regular: cost term is quadratic in input

Machine Model for Semiring Regular Functions

- Updates to registers must be copyless
  - Each register appears at most once in a right-hand-side
  - Update [x,y] := [min(x,y),y] not allowed
  - Necessary to maintain "linear" growth

- Need ability to simulate substitution
  - Register x carries two values c and d
  - Stands for the parameterized expression min(c, ?)+d
  - Besides min and inc, can substitute ? with a value

- Resulting model coincides with regular functions over semiring

- Open: Decidability of equivalence over (N, min , +c)

Discounted Cost Regular Functions

- ☐ Basic element: (cost c, discount d)
- ☐ Discounted sum: $(c_1,d_1)*(c_2,d_2) = (c_1+d_1c_2, d_1d_2)$
- ☐ Example of non-commutative monoid
- ☐ Classical Model: Future discounting
  - Cost of a path: $(c_1,d_1) * (c_2,d_2) * \ldots * (c_n,d_n)$
  - Polynomial-time algorithm for "generalized" shortest path
- ☐ Past discounting
  - Cost of a path: $(c_n,d_n) * (c_{n-1},d_{n-1}) * \ldots * (c_1,d_1)$
  - Same PTIME algorithm works for shortest paths
- ☐ Prioritized double discounting
  - Cost = $(c_1,d_1) * \ldots * (c_n, d_n) * (c'_1,d'_1) * \ldots * (c'_n,d'_n)$
  - Shortest path: NExpTime algorithm
- ☐ Open: Shortest path for Discounted Cost Register Automata

Conclusions

- Streaming String Transducers and Cost Register Automata
  - Write-only machines with multiple registers to store outputs
- DReX: Declarative language for string transformations
  - Robust expressiveness with decidable analysis problems
  - Prototype implementation with linear-time evaluation
  - Ongoing work: Analysis tools
- Emerging theory of regular functions
  - Some results, new connections
  - Many open problems and unexplored directions

Acknowledgements and References

- Streaming String Transducers (with P. Cerny; POPL'11, FSTTCS'10)
- Transducers over Infinite Strings (with E. Filiot, A. Trivedi; LICS'12)
- Streaming Tree Transducers (with L. D'Antoni; ICALP'12)
- Regular Functions and Cost Register Automata
  (with L. D'Antoni, J. Deshmukh, M. Raghothaman, Y. Yuan; LICS'13)
- Decision problems for Additive Cost Regular Functions
  (with M. Raghothaman; ICALP'13)
- Infinite-String to Infinite-Term Regular Transformations
  (with A. Durand, A. Trivedi; LICS'13: Next session)
- Min-cost problems for Discounted Sum Regular Functions
  (with S. Kannan, K. Tian, Y. Yuan; LATA'13)
- Regular combinators for string transformations
  (with A. Freilich and M. Raghothaman, LICS'13)
- DReX (with L. D'Antoni and M. Raghothaman; POPL'15)